Abstract

The relations among a given set of observables on a quantum system are effectively captured by their so-called joint numerical range, which is the set of tuples of jointly attainable expectation values. Here we explore geometric properties of this construct for Pauli strings, whose pairwise commutation and anticommutation relations determine a graph G. We investigate the connection between the parameters of this graph and the structure of minimal ellipsoids encompassing the joint numerical range, and we develop this approach in different directions. As a consequence, we find counterexamples to a conjecture by de Gois et al. [Phys. Rev. A 107, 062211 (2023)], and answer an open question raised by Hastings and O’ Donnell [STOC 2022: Proceedings of the 54th Annual ACM SIGACT Symposium on Theory of Computing, pp. 776–789], which implies a new graph parameter that we call “β(G)”. Furthermore, we provide new insights into the perennial problem of estimating the ground-state energy of a many-body Hamiltonian. Our methods give lower bounds on the ground-state energy, which are typically hard to come by, and might therefore be useful in a variety of related fields.