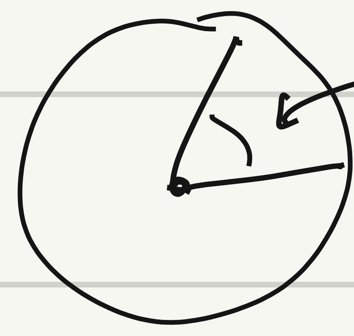
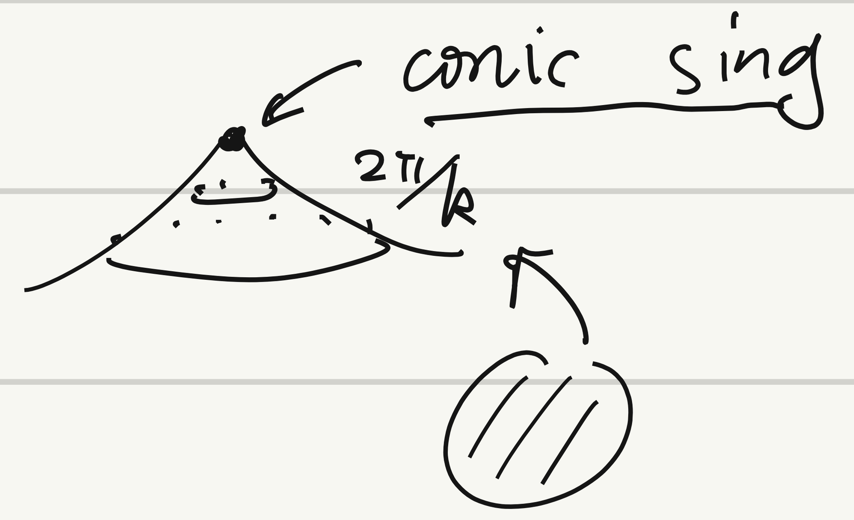
$\Gamma < PSL(2,\mathbb{R}) \cong Isom^+(\mathbb{H})$ Fuchsian (finitely generated)

– with torsion $\exists A \in \Gamma \ \exists k \in \mathbb{N}_{>0},\ A^k = id$



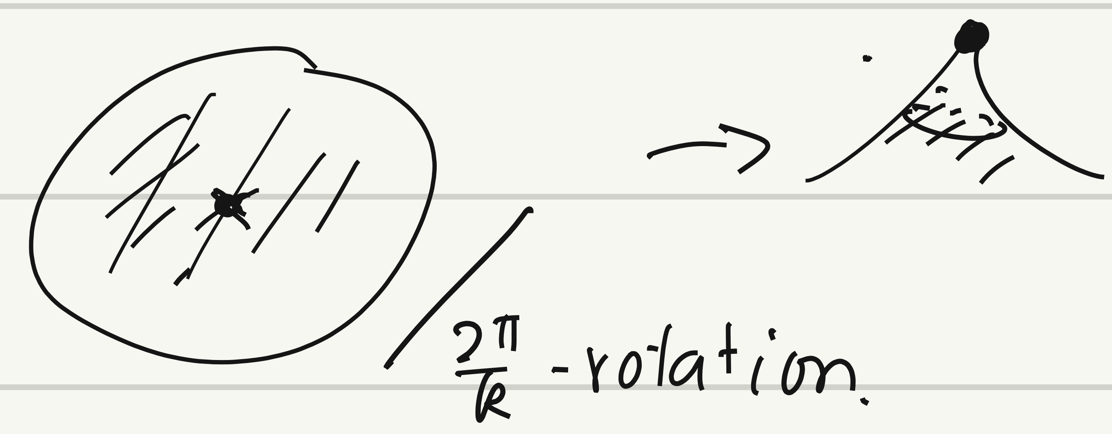
$\frac{2\pi}{k}$ rotation



– without torsion.

$\mathbb{H}/\Gamma$ → orbifold. (with conic sing)

$\mathbb{H}/\Gamma$ → surface (manifold)



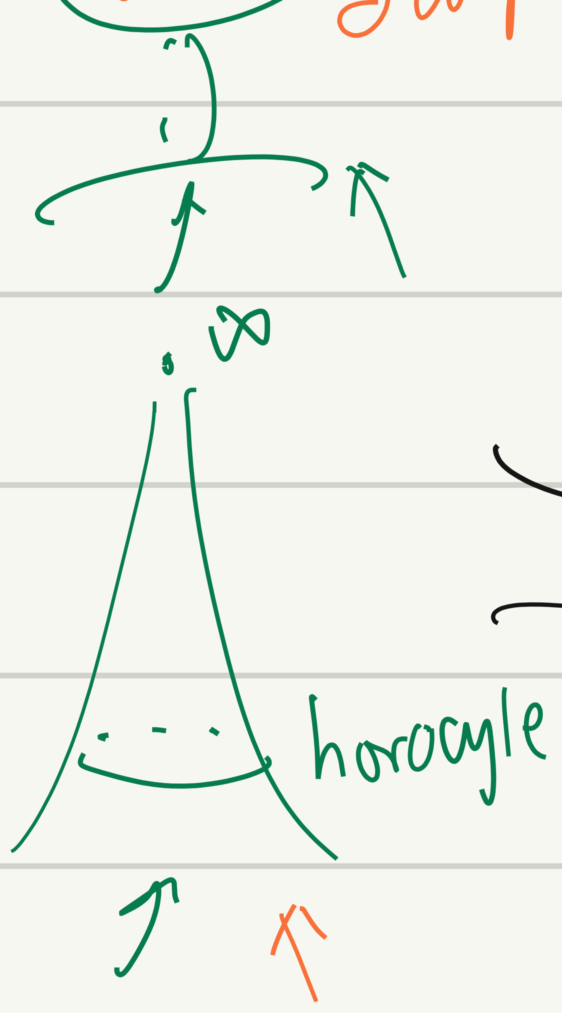
$\Sigma_{g,n}$ genus # of ∂ component

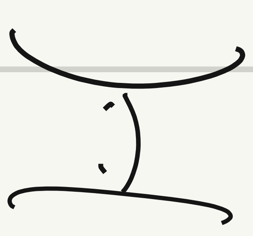
hyp, para, hyp, Torsion

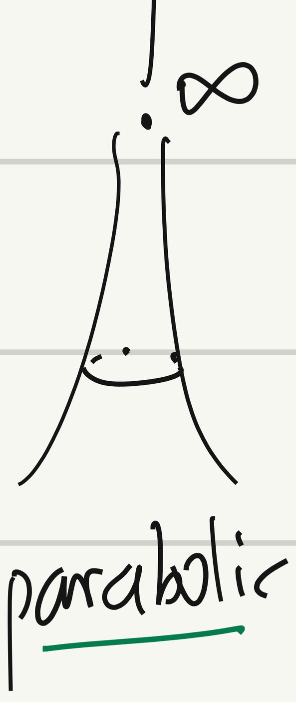
$S(g, n, b, c + \ldots)$ ∃ finite order elt.

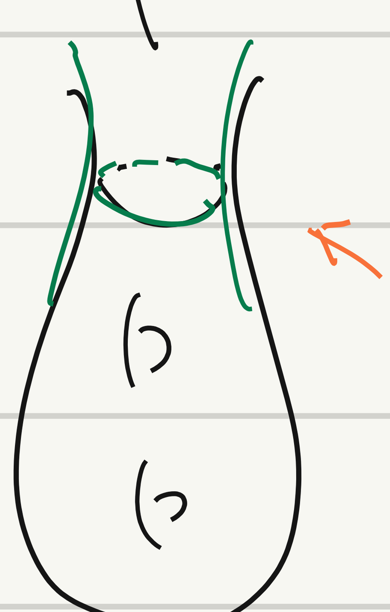
genus, cusp, Funnel boundary, conic sing.

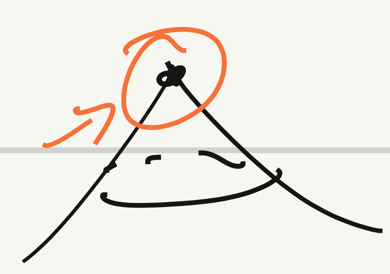
∃ hyp metric $\chi(S) < 0$.

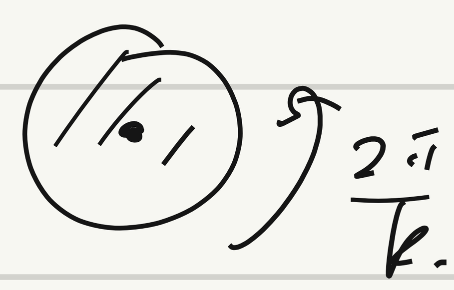


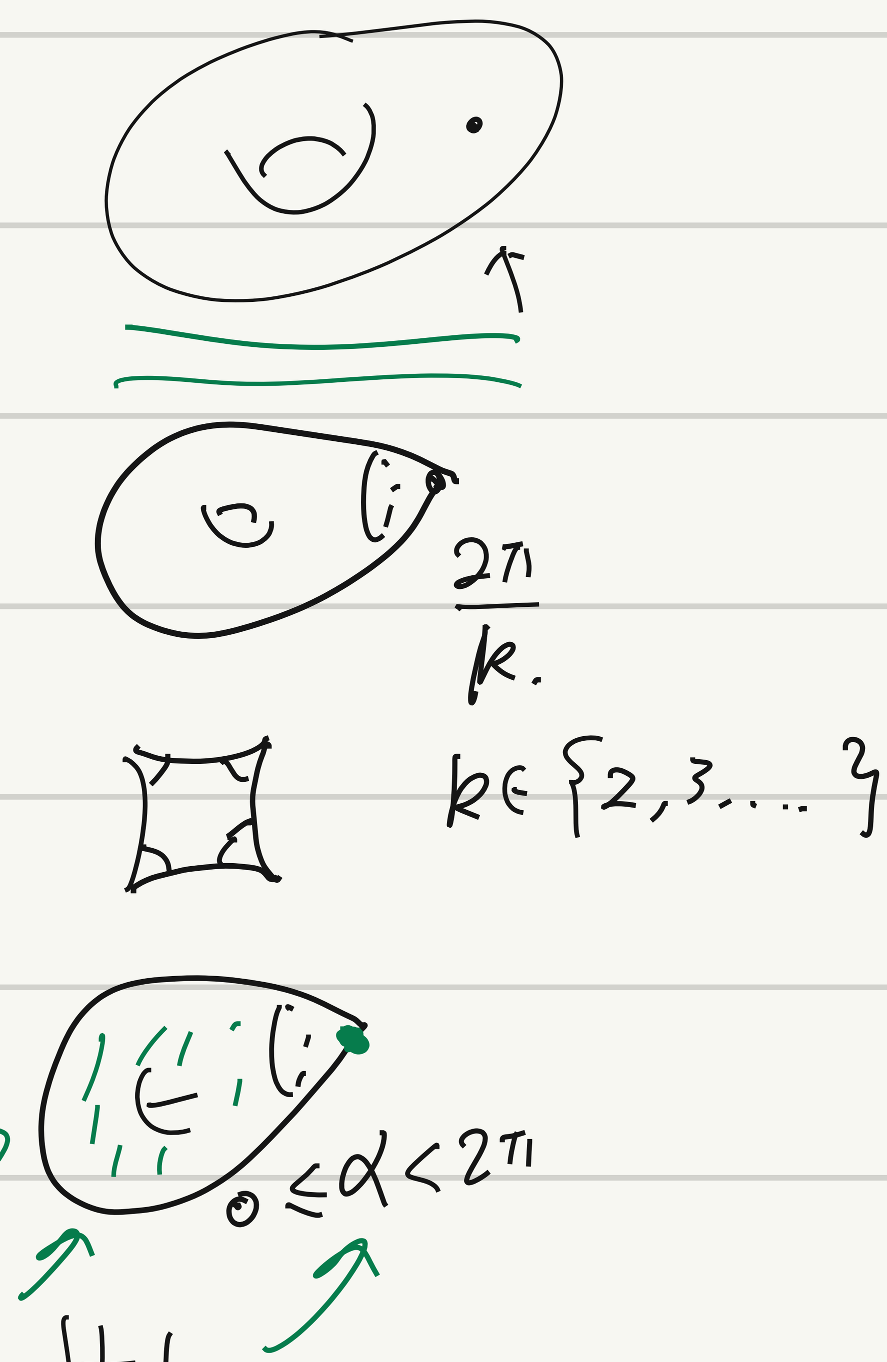


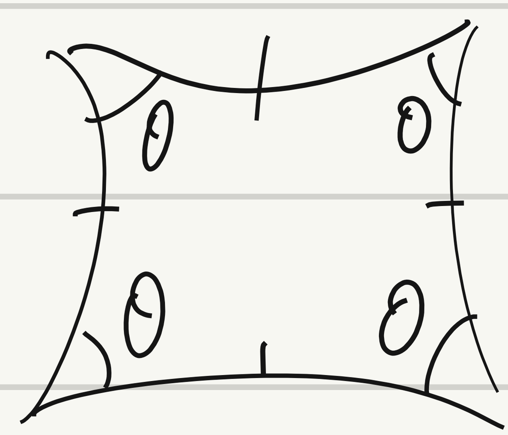


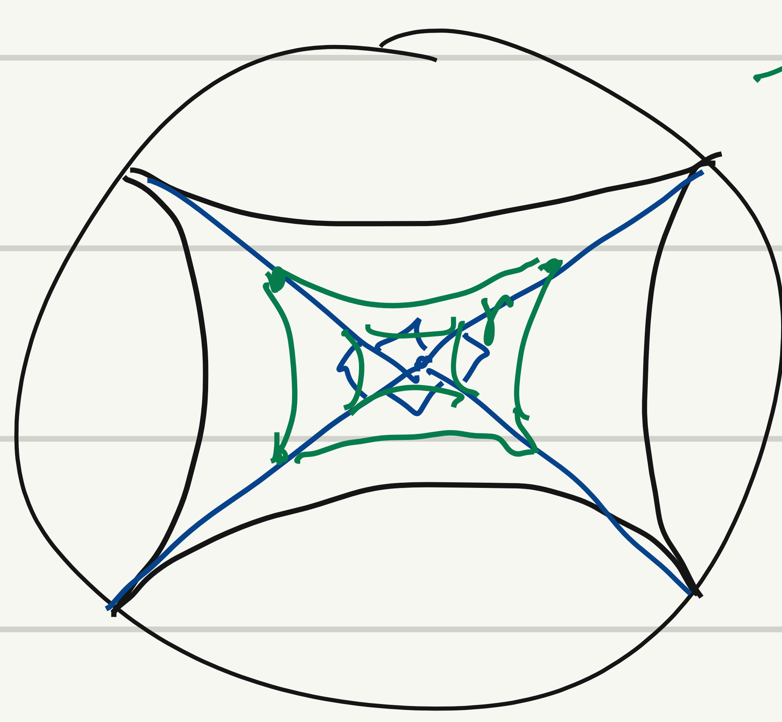






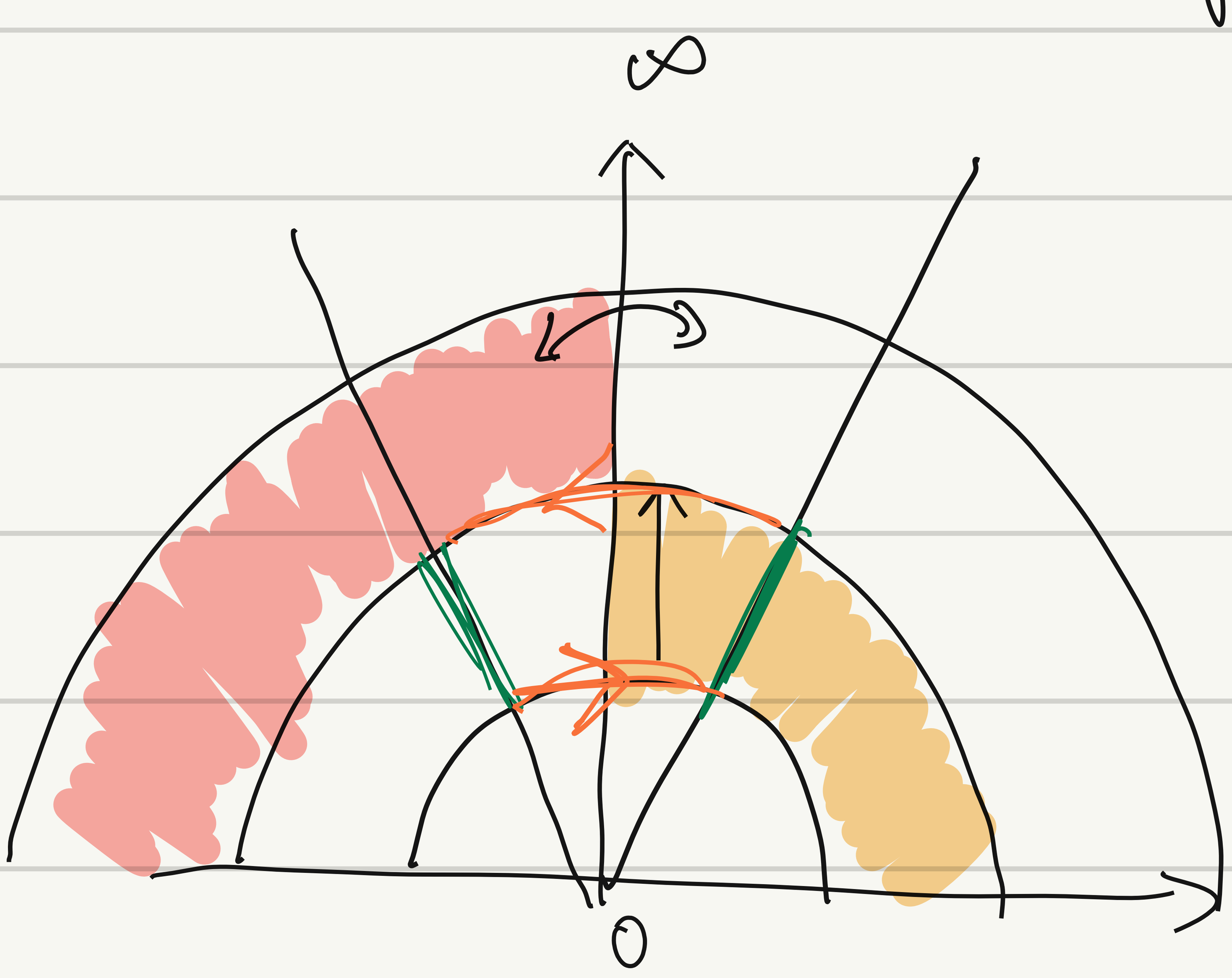






$2-2g-n$

$= 2-2\times1-1 = -1<0$

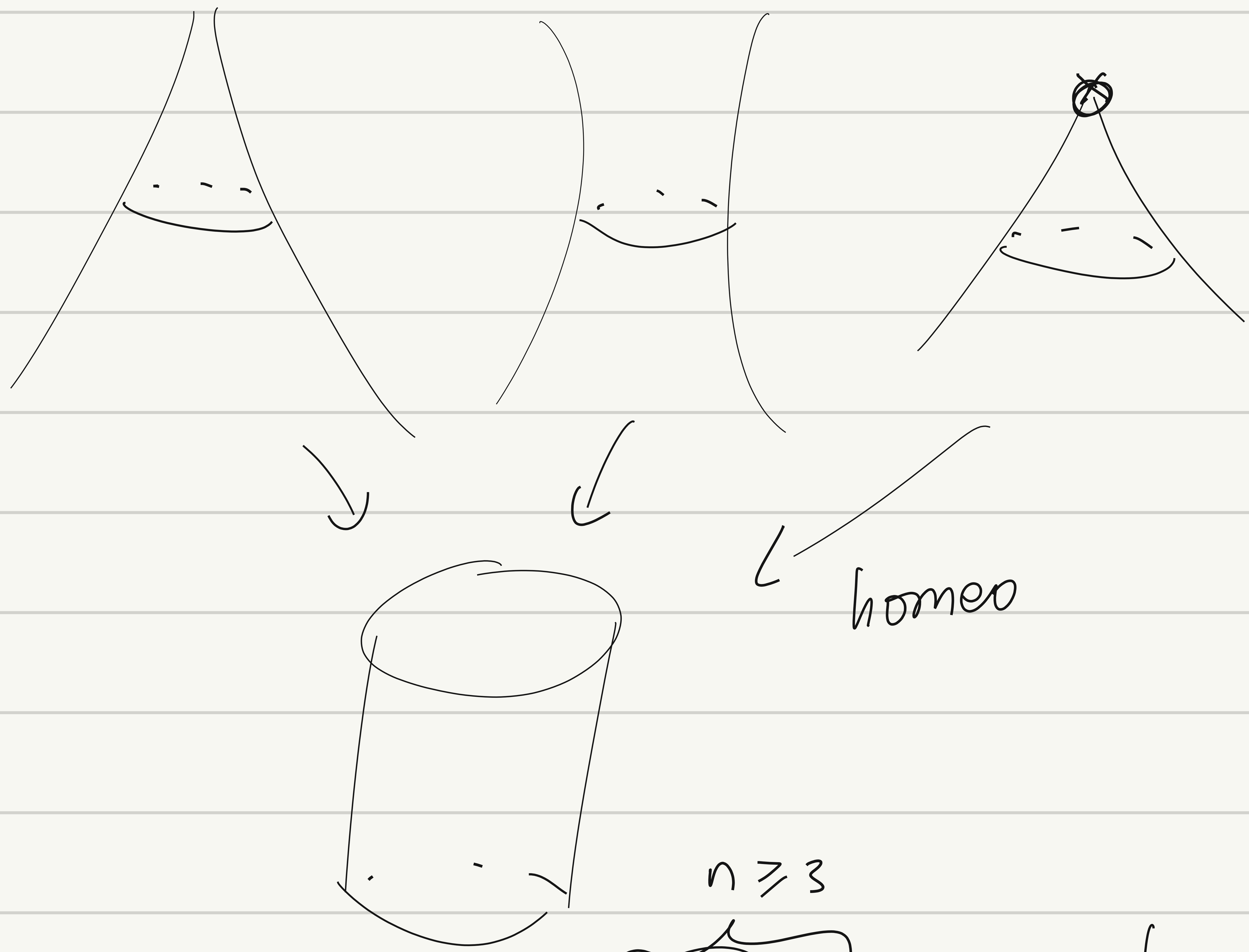


$r \to 0 \quad \theta \to \frac{\pi}{2}$

$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{bmatrix}(z) = \lambda^2 z$

$f(z) = -\lambda^2 \bar{z}$

reverse orientation of $\mathbb{H}$.

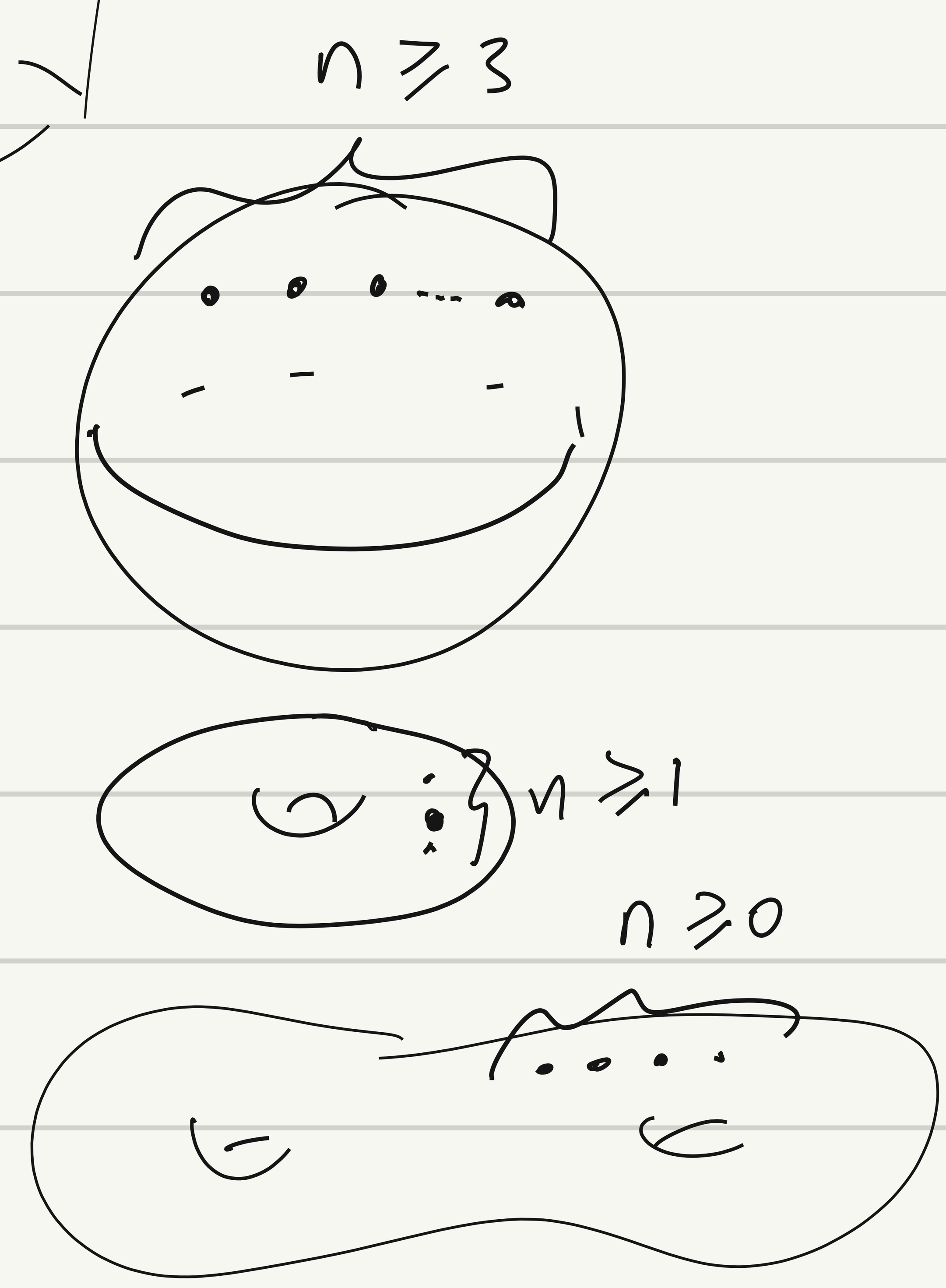


$\Sigma_{g,n}$.

$g=0 \quad n\geq 3$

$g=1 \quad n\geq 1$

$g\geq 2 \quad n\geq 0$



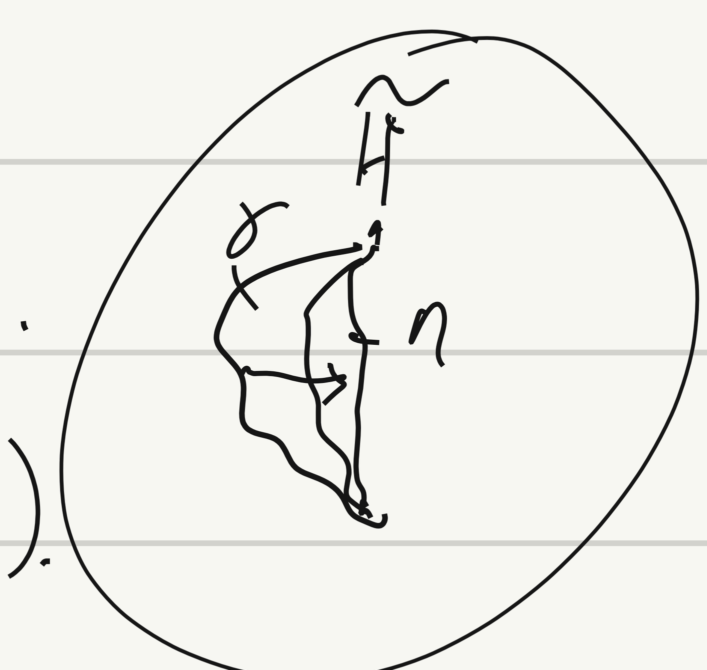
$\pi_1(\Sigma_{g,n}) \ncong \mathbb{Z}$.

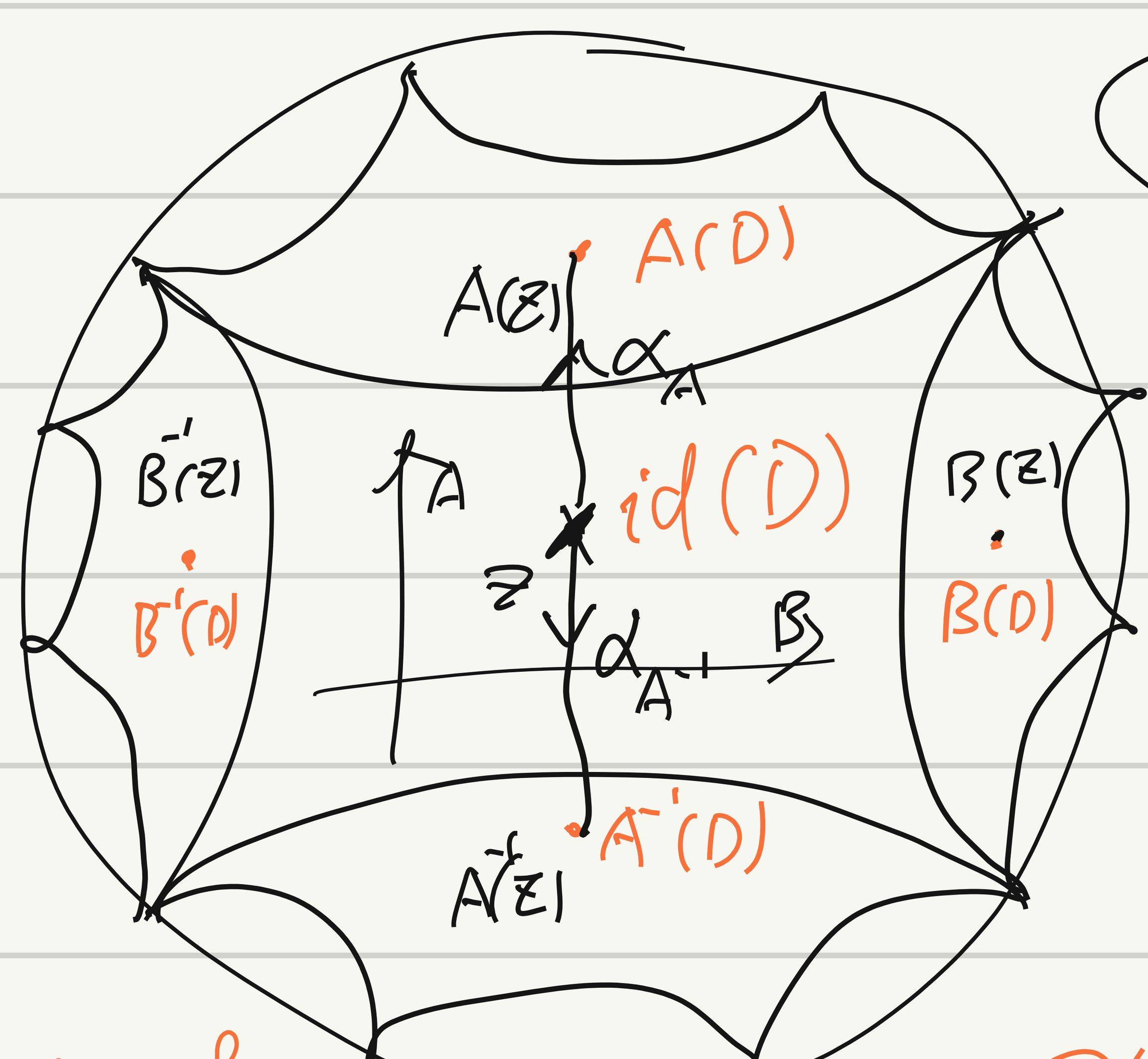
$\chi(\Sigma_{g,n}) = 2-2g-n<0$

$g=0$	$2-n<0$	$n>2$
$g=1$	$-n<0$	$n>0$
$g\geq 2$	$n>\underbrace{2-2g}_{<0}$	$n\geq 0$.

Γ torsion free.

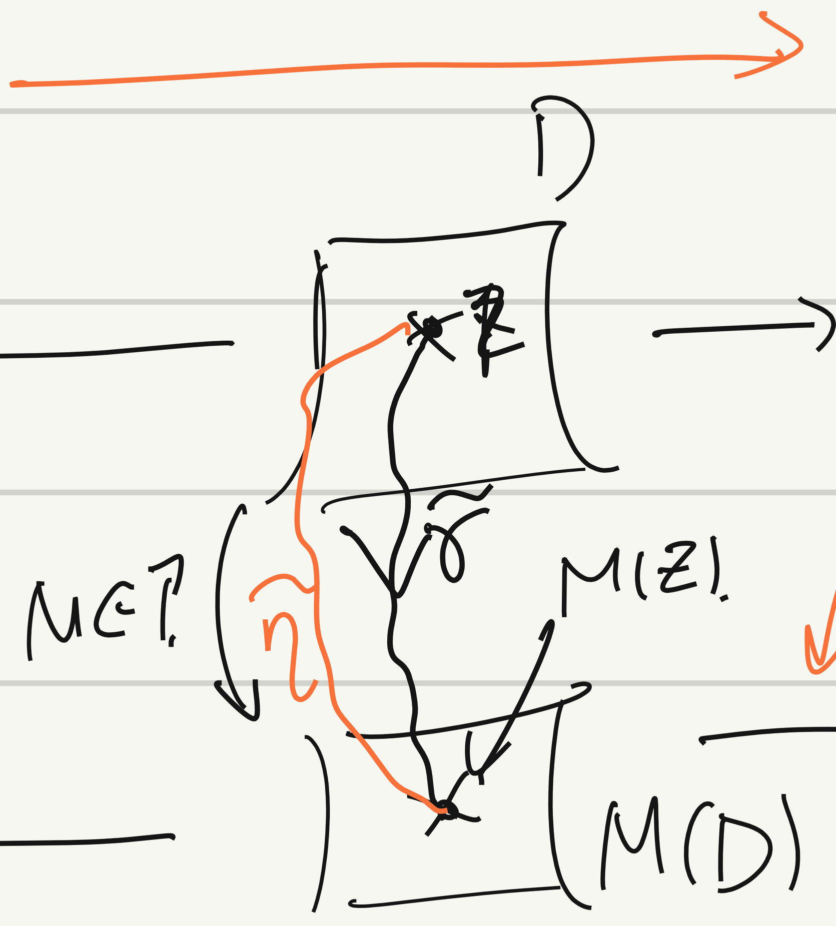
$\Gamma \cong \pi_1(\mathbb{H}/\Gamma)$.





$z,\ M(z) \in [z]_\Gamma$

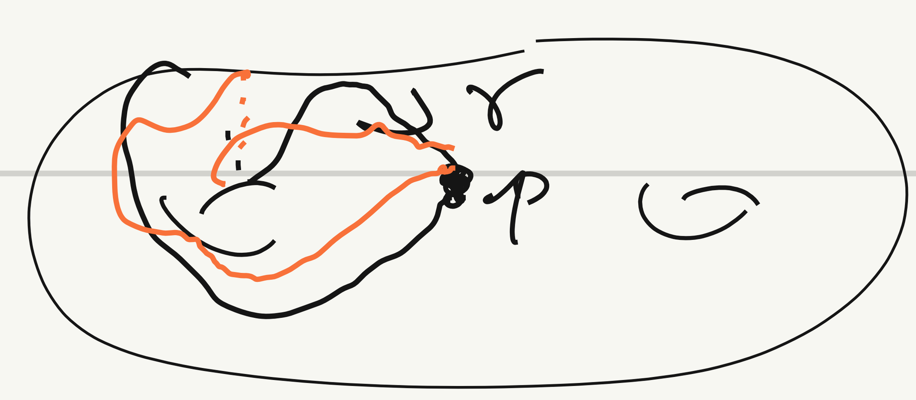
$\Gamma \longrightarrow \pi_1(\mathbb{H}/\Gamma)$.



$id \longleftarrow \qquad \longrightarrow id$

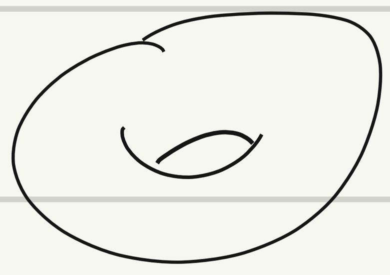
indep of choice of $\tilde{\gamma}$

$M \longleftarrow \qquad M(D) \longrightarrow [\gamma]_p$.

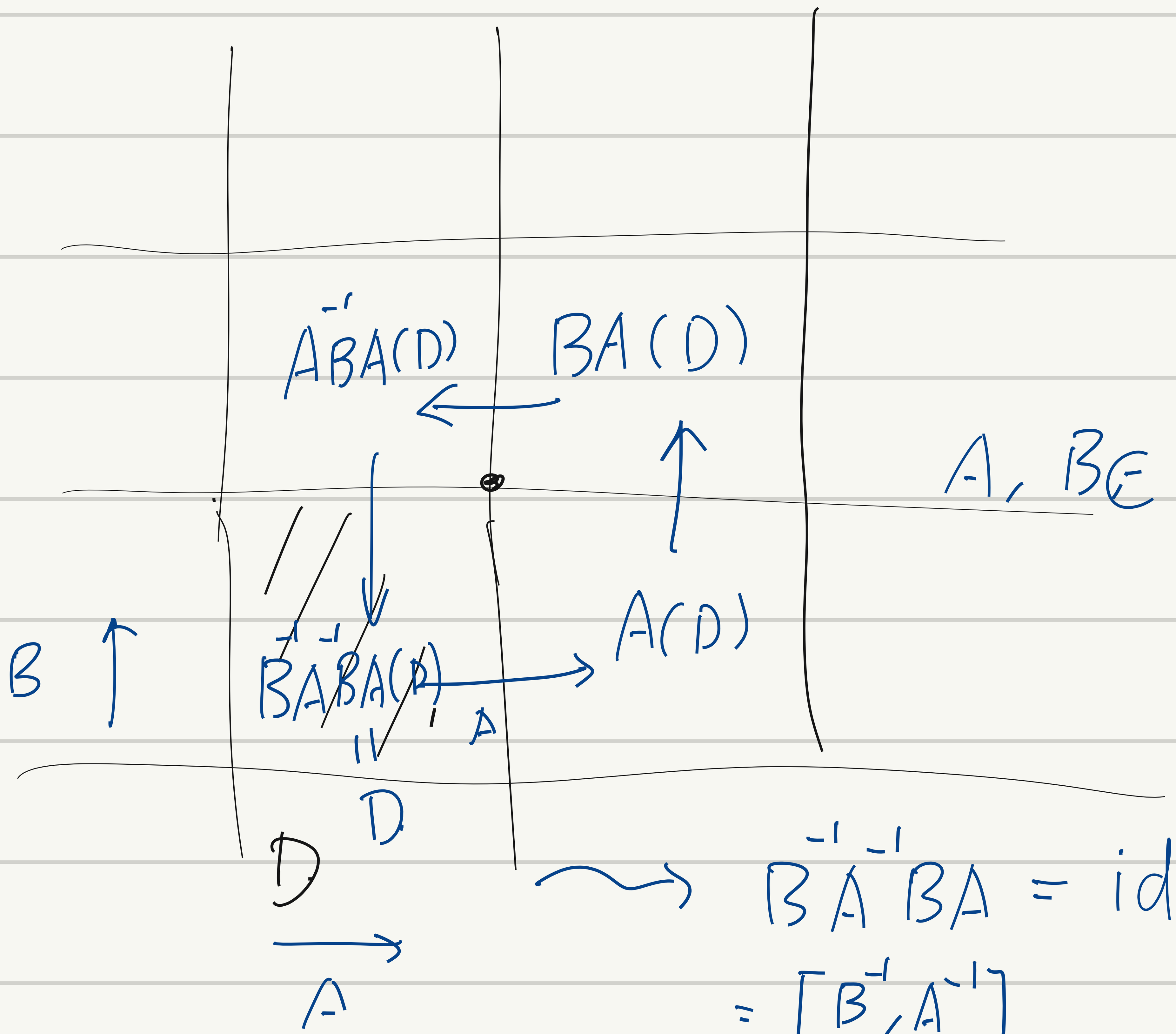


$\tilde{\gamma} \sim \tilde{\eta}$ in $\mathbb{H}$

$\gamma \sim \eta$ in S



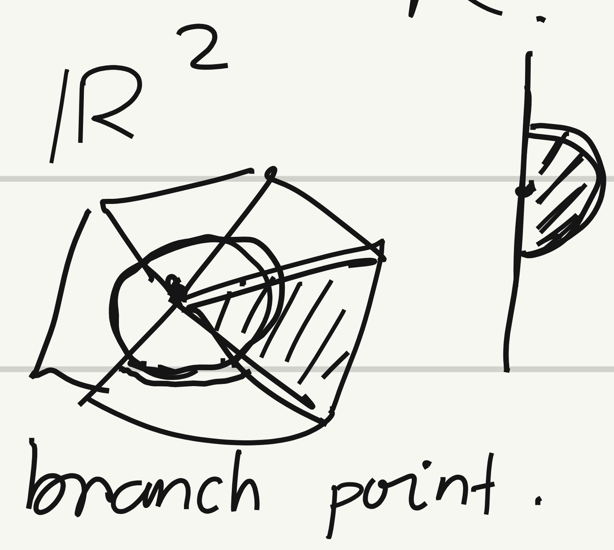
$$\pi_1(T) \cong \mathbb{Z}^2 \cong \langle a, b \mid \underbrace{[a,b] = id}_{R} \rangle$$

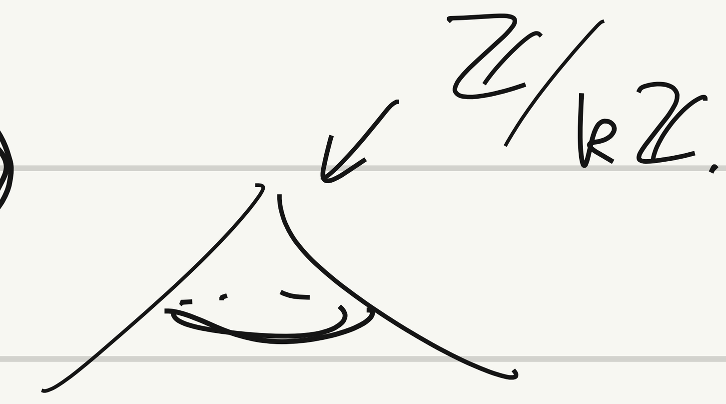


$\rightsquigarrow B^{-1}A^{-1}BA = id.$

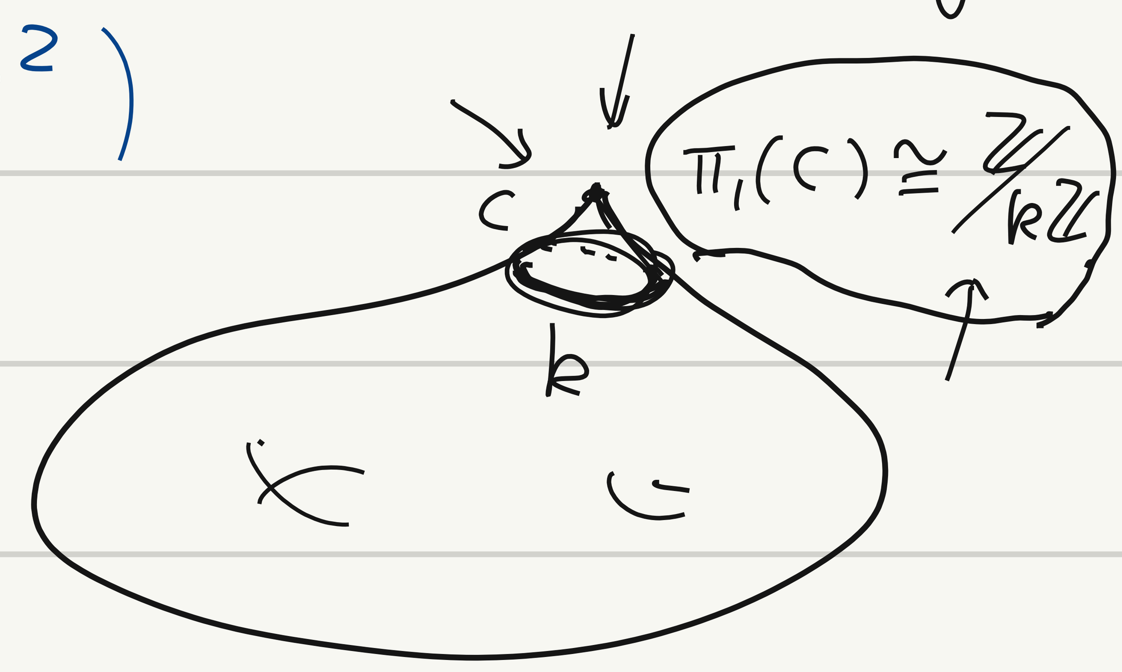
$= [B^{-1}, A^{-1}]$

$A, B \in \mathrm{Isom}(\mathbb{R}^2)$

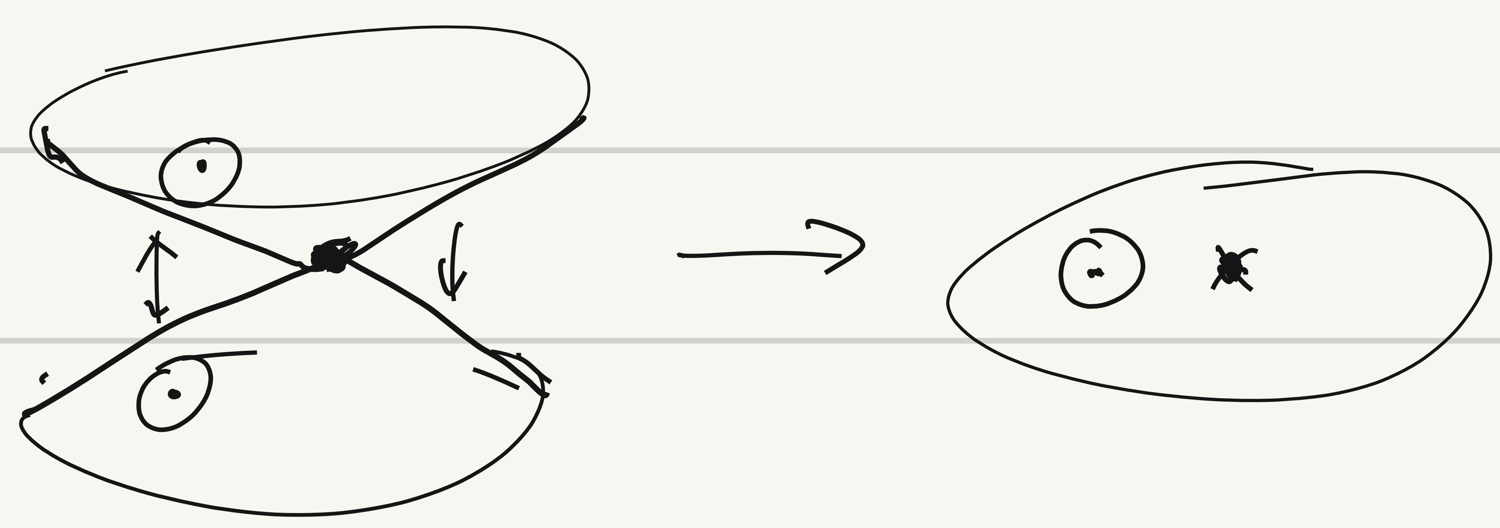


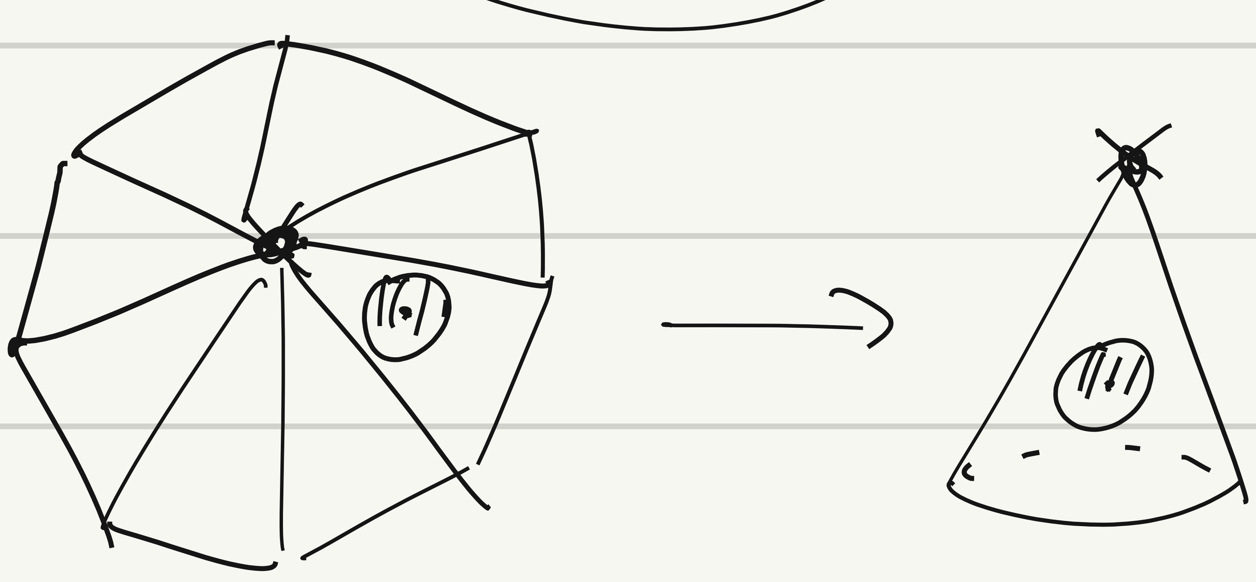


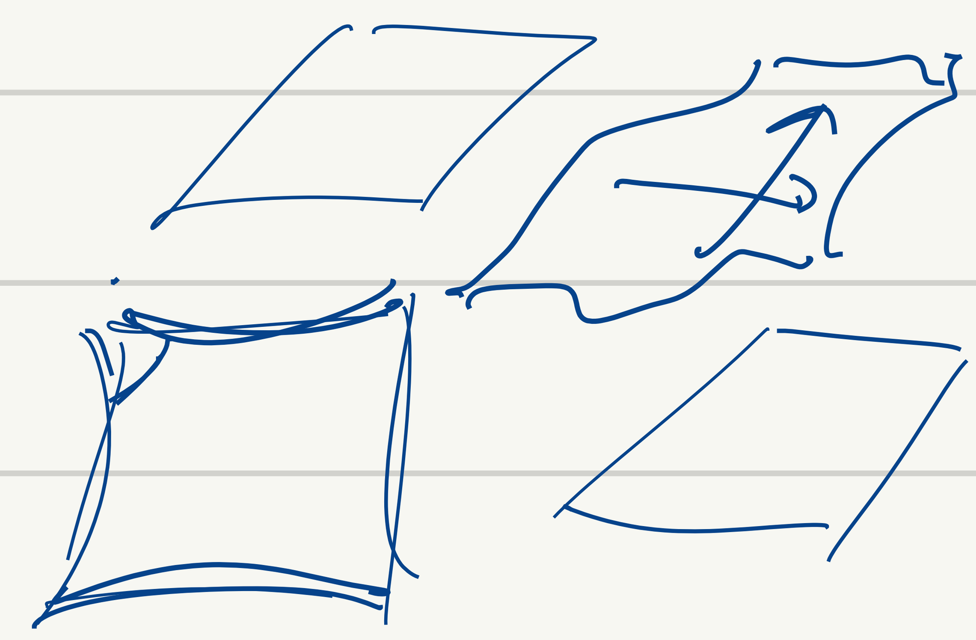
conic sing

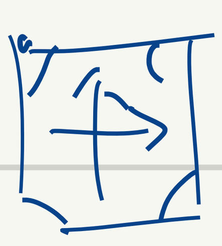


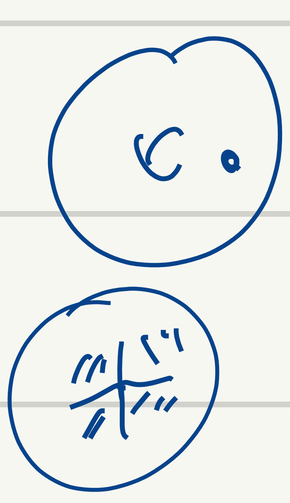
$T \longrightarrow \pi_1(\mathbb{H}/T)$

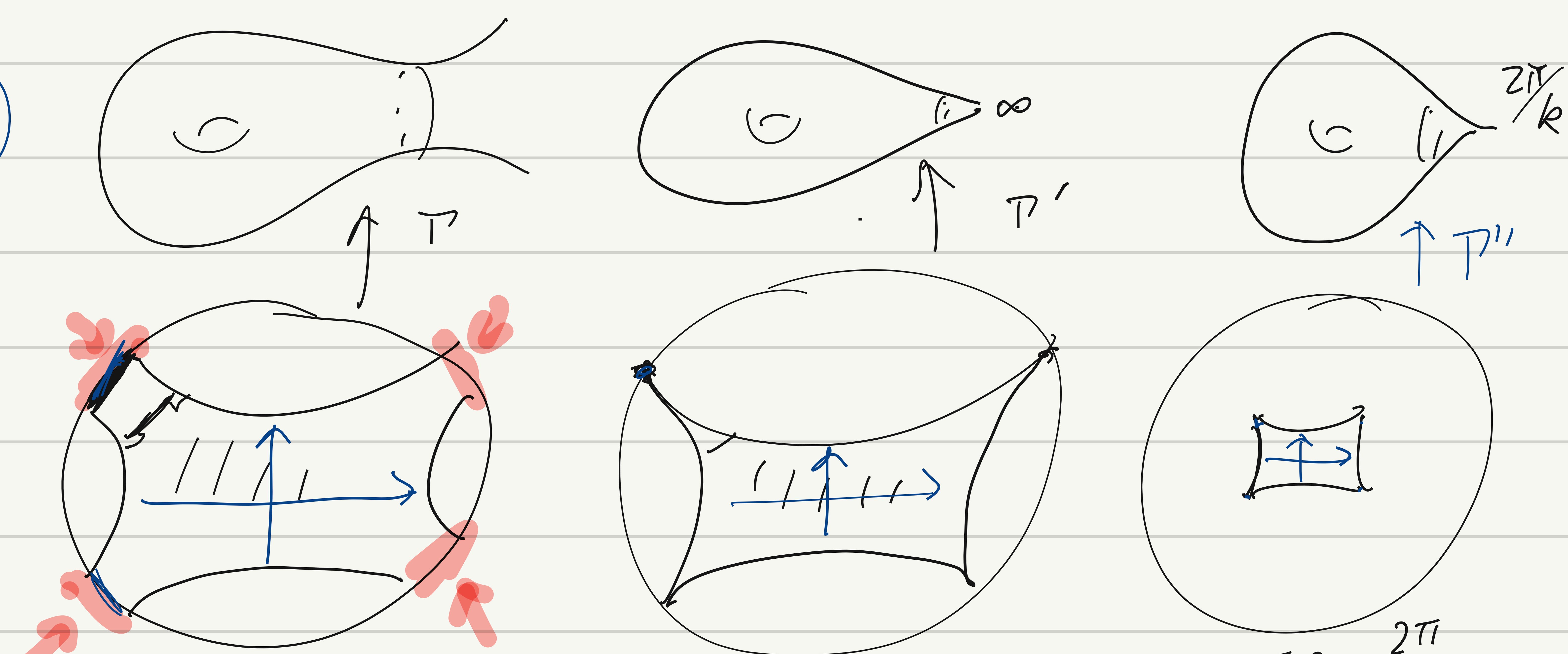








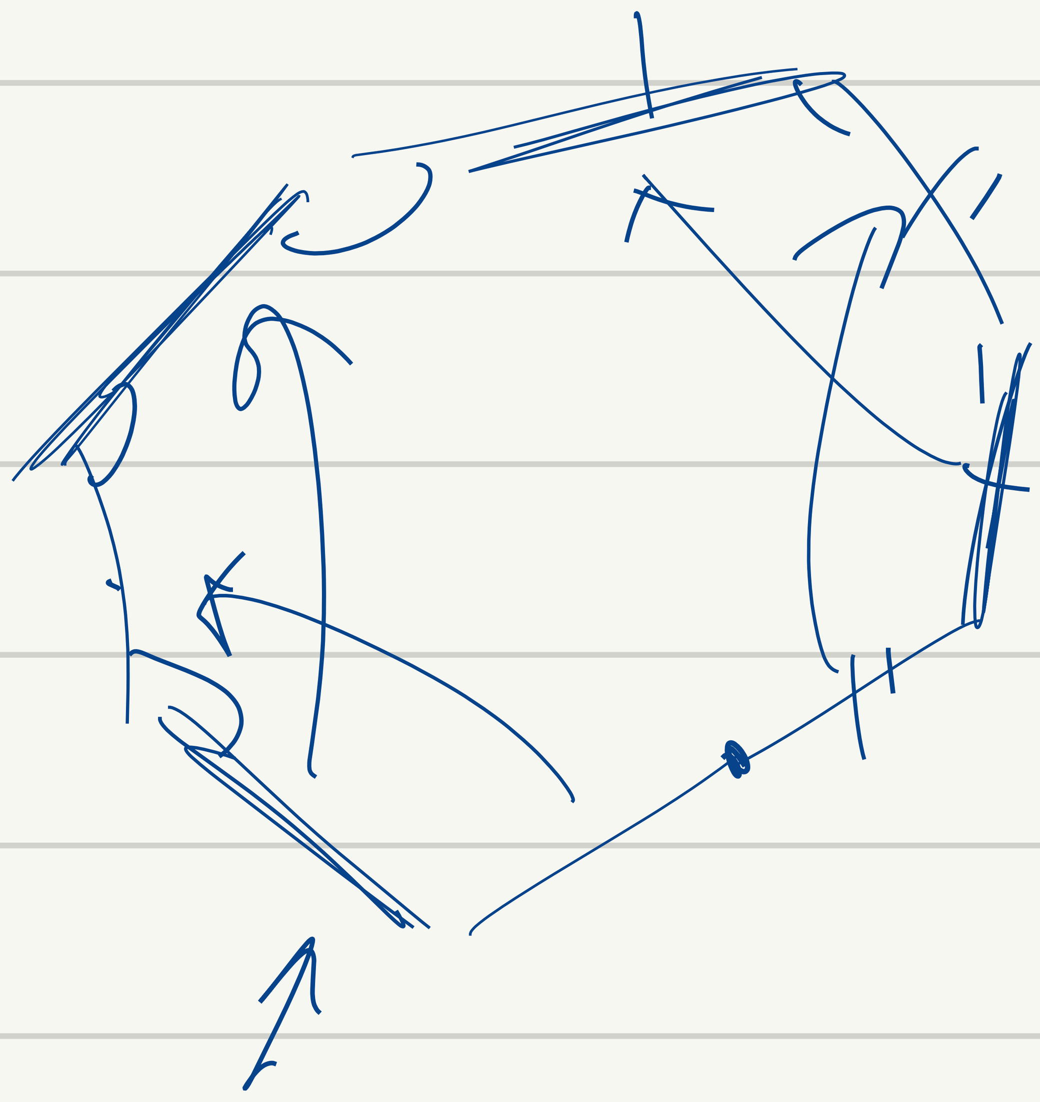


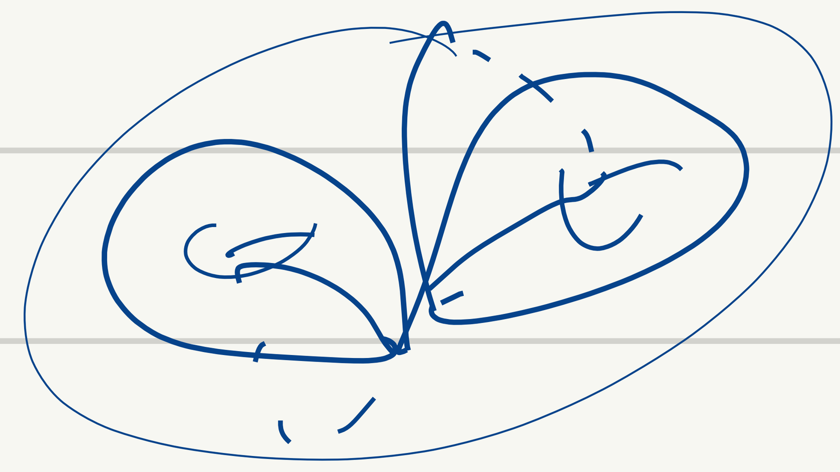


$\Sigma \theta_i = \frac{2\pi}{k}.$

$T \cong \pi_1(S)$ $\quad T \quad (\pi_1(S), \rho : \pi_1(S) \to PSL(2, \mathbb{R}))$

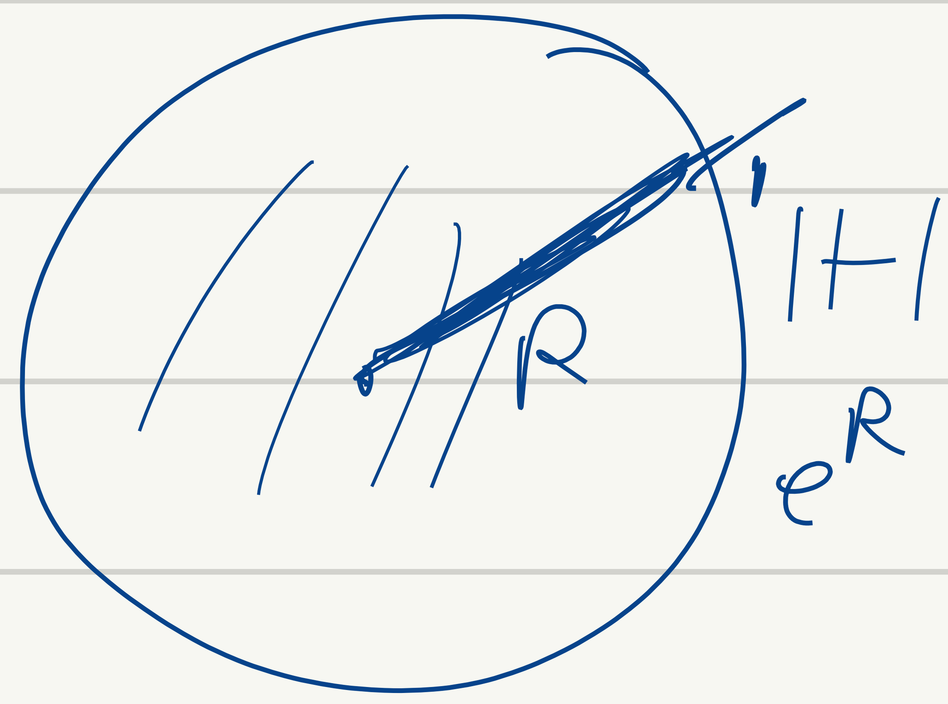
$= \mathrm{Im}(\rho)$

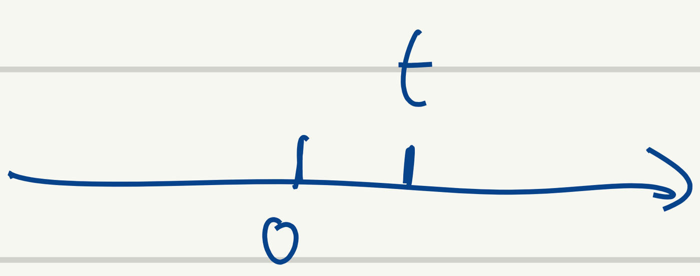




$$\operatorname{ch} R = \frac{e^{R} + e^{-R}}{2}$$

$e^{-R} \to 0,\ R \to \infty$





$R \gg 0.$ $\quad \operatorname{ch} R \sim \dfrac{e^{R}}{2}.$

R^2

$(R+1)^2 - R^2$

$e^{R+1} - e^{R}.$