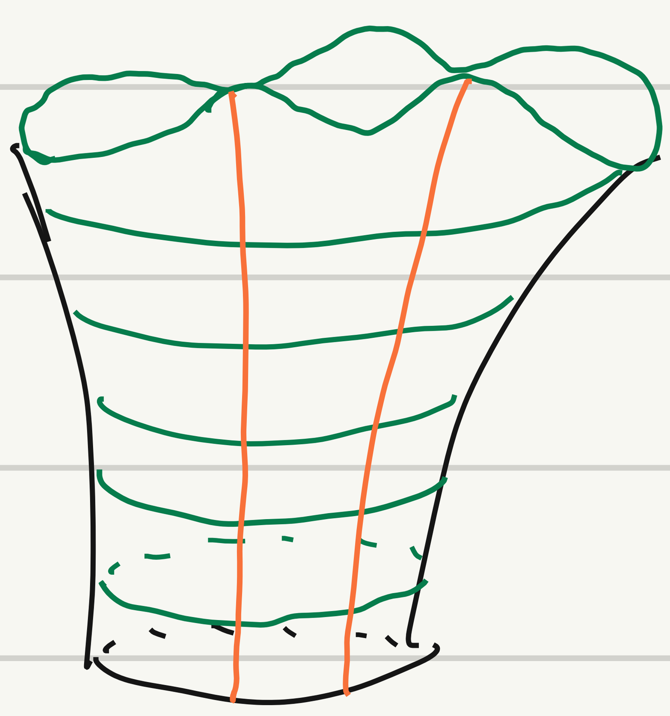
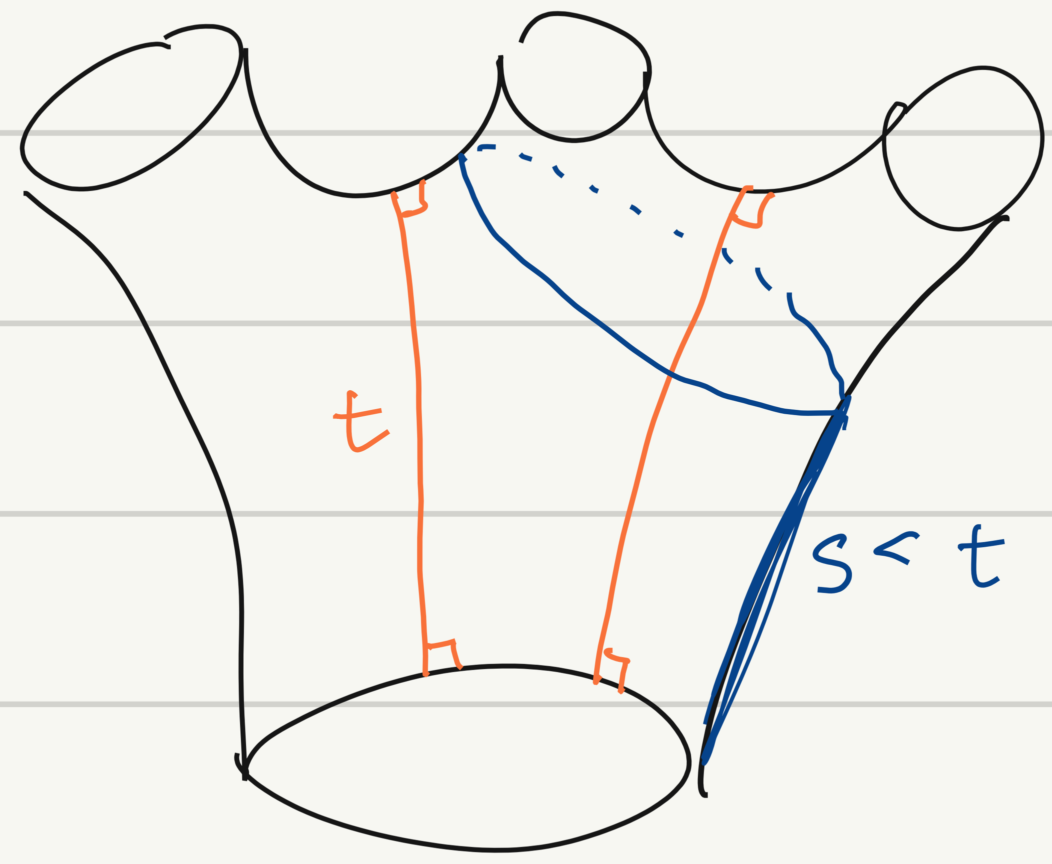
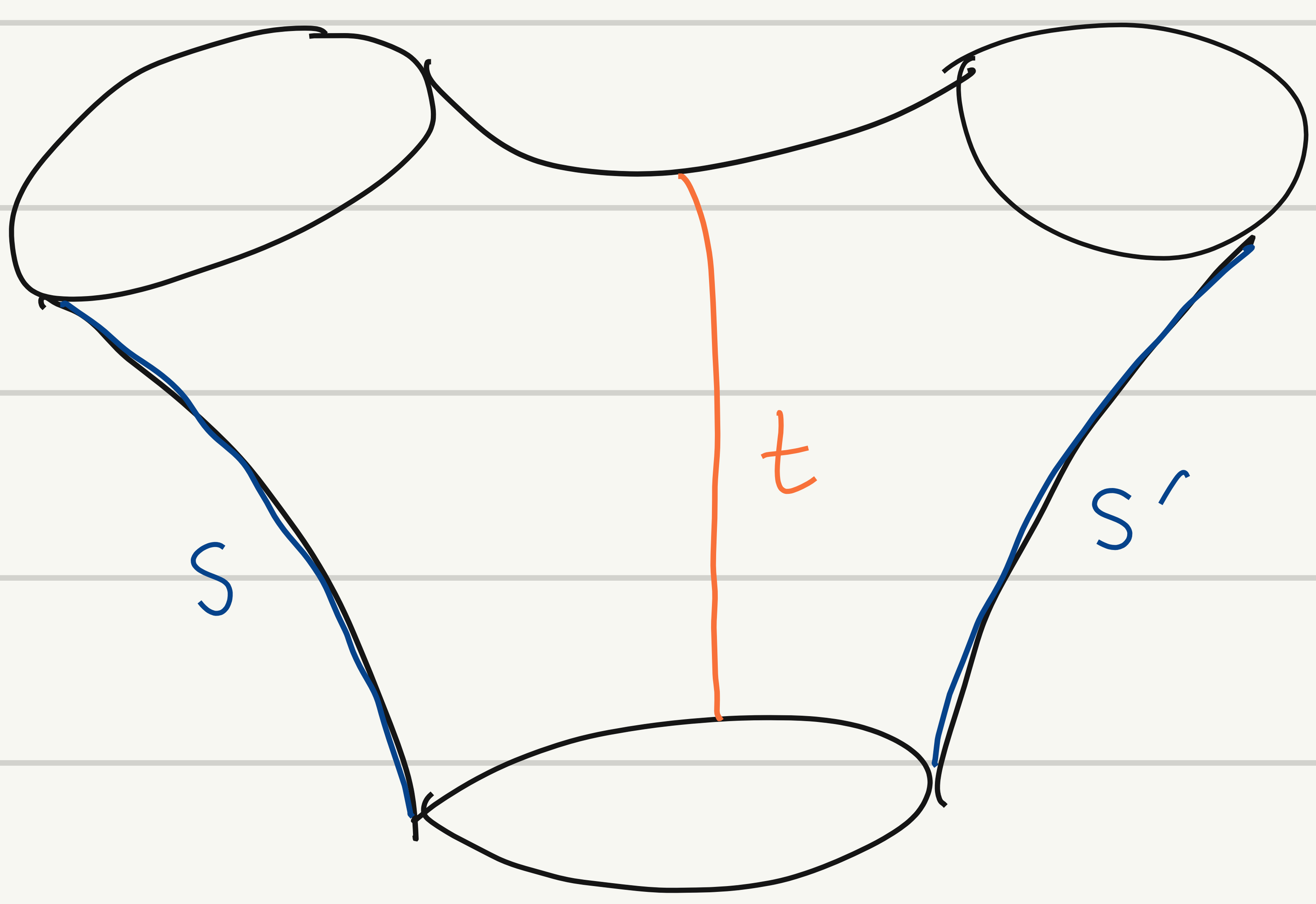
<u>Collar lemma</u>: Let $\ell(\gamma)$ be the length of a simple closed geodesic γ in S. Then $\exists$ a collar of γ with width $\operatorname{arcsinh} \frac{1}{\sinh \frac{\ell(\gamma)}{2}}$.

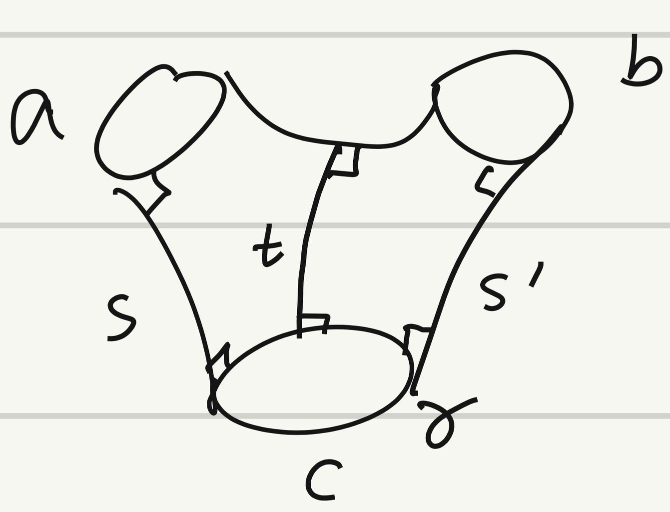
<u>Proof</u>:





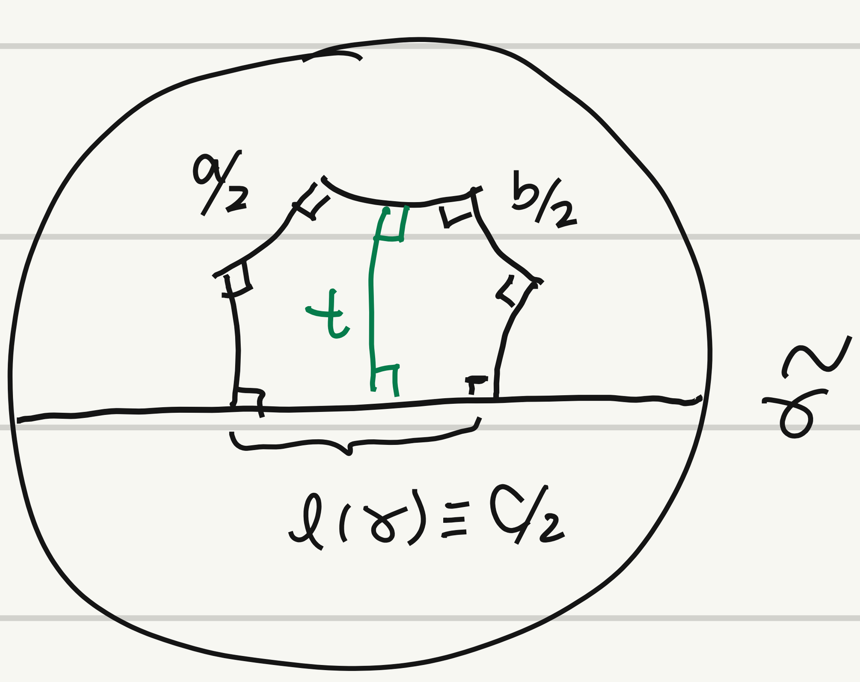
Hence it is enough to consider only in a pair of pants



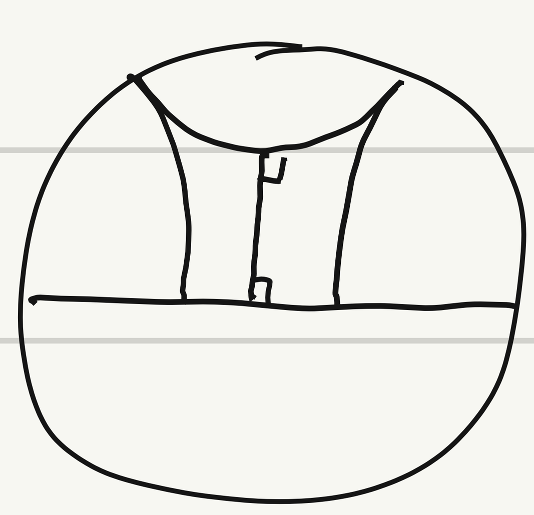


Find a lower bound of $\min\{t, s, s'\}$ for a fixed $\ell(\gamma)$. and all possible $a, b \in \mathbb{R}_{\geq 0}$

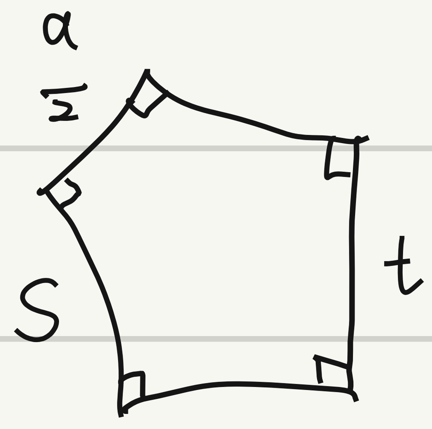
It is enough to study the right angled hexagon.



<u>Lemma</u>: The minimum of t is realized by $P(0,0,c)$

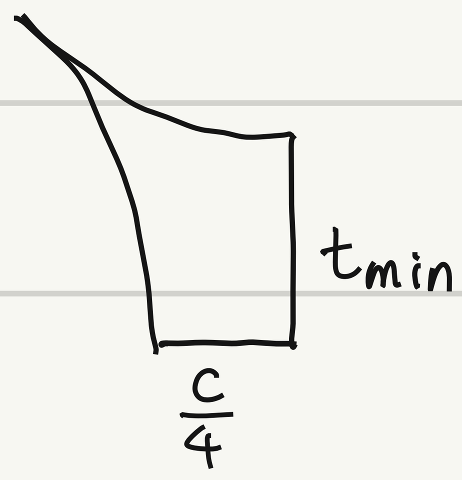


Proof:



$\sinh\frac{a}{2}\sinh s = \cosh t$ (trigonometry formula for right angled pentagon)

t is strictly increasing as a function of s.



$\sinh\frac{c}{4}\sinh t_{min} = 1$ (trigonometry formula for quadrilateral of angle $(\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2},0)$)

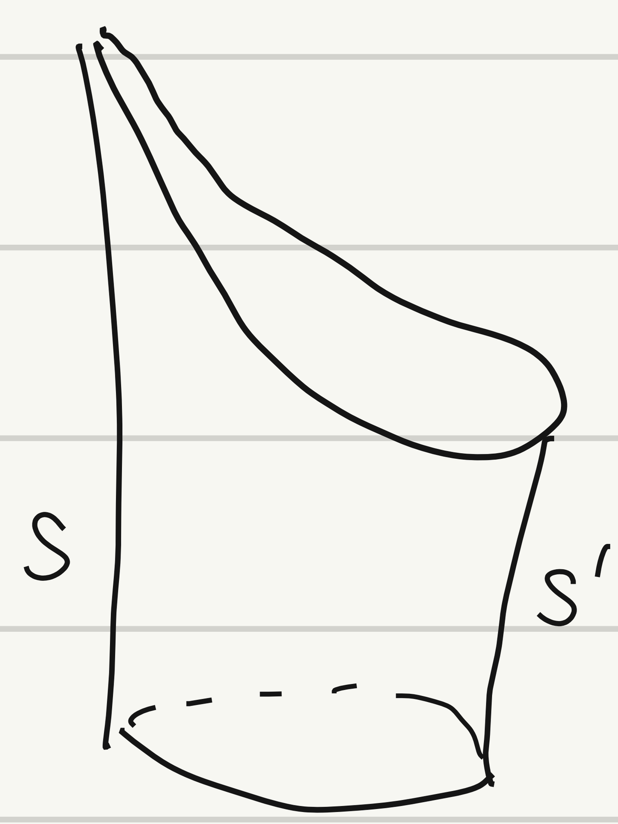
$t_{min} = \operatorname{arcsinh}\frac{1}{\sinh\frac{c}{4}}$

Moreover $s = s' = \infty > t_{min}$.

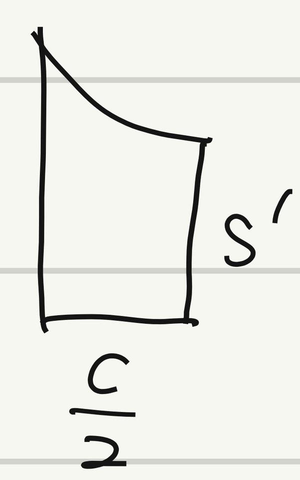
<u>Lemma</u> the minimum of s and s' is realized by $P(0,\infty,c)$

$a \to 0$

$b \to \infty$

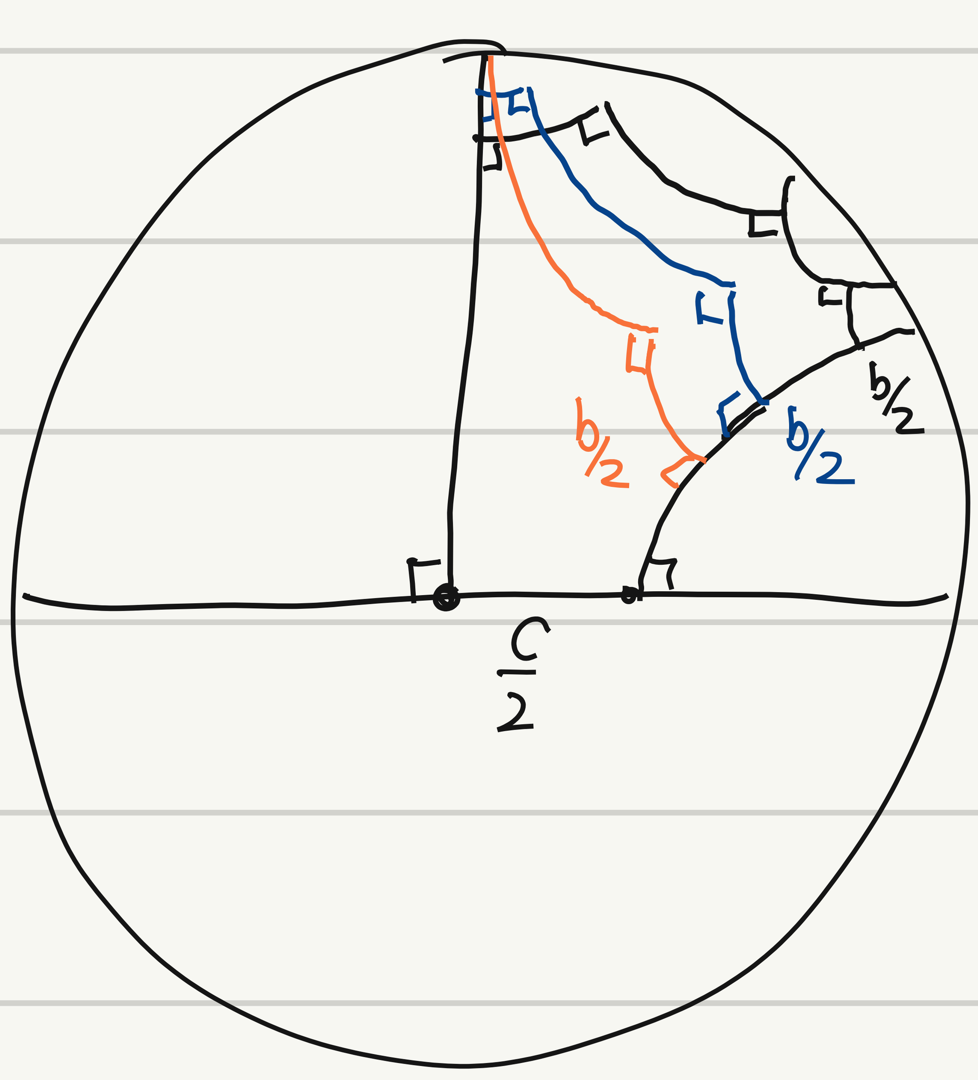


$\min s' = \operatorname{arcsinh}\frac{1}{\sinh\frac{c}{2}}$



Proof: $\cosh s' = \dfrac{\cosh\frac{c}{2}\cosh\frac{b}{2} + \cosh\frac{a}{2}}{\sinh\frac{c}{2}\sinh\frac{b}{2}} \overset{a\searrow 0}{\geq} \dfrac{\cosh\frac{c}{2}\cosh\frac{b}{2}}{\sinh\frac{c}{2}\sinh\frac{b}{2}} \overset{b\nearrow\infty}{\geq} \dfrac{\cosh\frac{c}{2}}{\sinh\frac{c}{2}}$ ∎

(trigonometry formula for a right angled hexagon.)



Hence $\min\{s, s', t\} > \operatorname{arcsinh}\dfrac{1}{\sinh\frac{c}{2}} = \operatorname{arcsinh}\dfrac{1}{\sinh\frac{\ell(\gamma)}{2}}$

$\Rightarrow C_\gamma\left(\operatorname{arcsinh}\frac{1}{\sinh\frac{\ell(\gamma)}{2}}\right)$ is a collar of γ. ∎