EXERCISE SHEET #2

Prove the uniform boundedness of k-centers in δ-hyperbolic space:

Exercise 0.1. (1) *Let $\Delta = \Delta(abc)$ be a geodesic triangle with vertices $a, b, c \in X$ and o be a k-center for $k > 0$. Then*

$$d(c, o) - 2k \leq (a, b)_c \leq d(c, o) + k.$$

(2) *Let p, q be be two k-taut paths in X with same endpoints x and y. Let $z \in p, w \in q$ be two points such that $d(z, x) = d(w, x)$. Then*

$$d(z, w) \leq 2k + 16\delta.$$

(3) *Prove that the set of k-centers is of uniform diameter depending only on k and δ. Write the statement in a quantitative form and then prove it.*

Thin triangle property. Let $\Delta \subset X$ be a geodesic triangle in a metric space X. Let $\Delta' \subset T$ be a comparison triangle with same length of sides as those of Δ. There is a natural bijective map $\phi : \Delta \to \Delta'$ which sends sides of Δ isometrically to those of Δ'. We say that Δ is δ-*thinner* than Δ' for some $\delta \geq 0$ if for any two congruent points $x', y' \in \Delta'$, we have $d_X(\phi^{-1}(x'), \phi^{-1}(y)) \leq d_Y(x', y') + \delta$.

Exercise 0.2. *Let (X, d) be a geodesic metric space with δ-thin triangle property. Then there exists a constant $\delta' > 0$ such that every geodesic triangle is δ'-thinner than a companion geodesic triangle in a tree.*