Polyadic algebraic structures and their applications

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Abstract

We develop the polyadic analogs of such algebraic structures as groups, rings and fields, and investigate their arity properties which do not exist in the binary systems. First, we generalize the Hosszu-Gluskin theorem giving the general form of polyadic multiplication to "q-deformed" case. We introduce a new polyadic analog of homomorphism, the "heteromorphism" which changes the arity. Using heteromorphisms, we introduce multiactions and exotic multiplace representations, giving matrix examples for ternary case. Then new polyadic integer numbers are defined, to be an analog of Z. They are representatives of a fixed congruence class forming a polyadic ring. As an application to the number theory, we generalize famous Diophantine equations to polyadic integer numbers and investigate analogs of the Lander-Parkin-Selfridge conjecture, Fermat's last theorem and the Tarry-Escott problem. The "secondary" congruence classes are defined, which allows us to build the polyadic finite fields with unusual properties: they can be zeroless, zeroless and nonunital, have several units, or all elements can be units, also there exist non-isomorphic polyadic finite fields of the same order and arity shape. None of them are possible in the binary case. It is conjectured, that every polyadic finite field should contain a certain prime polyadic field (constructed here), as a minimal subfield, which can be considered as a polyadic analog of the Galois field GP(p).