Abstract

Let $S=\{s_i\}_{i=1}^{\infty}$ be any given infinite sequence of positive integers (note that all the s_i are not necessarily distinct and not necessarily monotonic). Let $f(x)$ be a polynomial of nonnegative integer coefficients. For any integer $n\ge 1$, one lets $S_n:=\{s_1, ..., s_n\}$ and $H_f(S_n):=\sum_{k=1}^{n}\frac{1}{f(k)^{s_k}}$. When $f(x)=x$ and $s_i=1$ for all positive integers $i\ge 1$, then Theisinger proved in 1915 that $H_f(S_n)$ is not an integer if $n\ge 2$. In 1923, Nagell extended Theisinger's theorem by showing that if $s_i=1$ for all positive integers $i\ge 1$ and $f(x)=ax+b$ with $a\ge 1$ and $b\ge 0$ being integers, then $H_f(S_n)$ is not an integer if $n\ge 2$. In this talk, we will speak about recent progress on the integrality of the reciprocal power sum $H_f(S_n)$.