Abstract

To protect a body from overheating, engineers usually coat the body with a thin thermal insulator (such as a space shuttle or turbine engine blades painted by an insulator). In cell biology, some chemical substances do not react in the bulk region of the cell, but react (and diffuse) very fast in the membrane of the cell, which results in cell polarization and cell division. These phenomena can both be modeled by a diffusion equation on a body coated by a thin layer, where the diffusion rates in the thin layer could be very different from that in the bulk body. Mathematically, this is the challenging multiscale problem. From the viewpoint of numerical simulation, it is also time-consuming to simulate this problem, since one needs to discretize the layer to very thin meshes to deal with the geometry. An effective strategy is to identify the effective approximate problems on the bulk domain excluding the behavior of the solution on the thin layer. At this point, the limiting boundary condition on the boundary of the bulk is the so-called effective boundary condition (EBC). In this talk, we present some results and methods of how to characterize the EBC as the thickness of the thin coating shrinks and the diffusion rate of the coating tends to zero or infinity. One will

find that not only the usual Dirichlet, Neumann and Robin EBC can be derived, but also some exotic EBC and even dynamical EBC and bulk-surface equation can be derived. We will also discuss the lifespan of EBC.