Chern Institute of Mathematics, October 2007

Lectures proposed by Alain Hénaut

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Introduction to planar web geometry

1. *General introduction, basic tools and two fundamental results*

Planar d-webs $\mathcal{W}(d)$. Examples : algebraic webs and dual projective curves, implicit webs or the differential equation viewpoint. Abelian relations and the sharp bound $\pi_d = \frac{1}{2}(d-1)(d-2)$ for the rank of a $\mathcal{W}(d)$: the resonance differential system, the classic Poincaré-Blaschke method. Birth certificate of the planar web geometry : Thomsen's closure and Blaschke curvature for a $\mathcal{W}(3)$.

2. *Main problems and abelian relations from the implicit viewpoint*

Abel theorem for algebraic webs $\mathcal{L}_C(d)$ and Lie-Darboux-Griffiths theorem for linear webs $\mathcal{L}(d)$. Exceptional webs $\mathcal{E}(d)$ for $d \geq 5$: Bol's $\mathcal{B}(5)$ and the polylogarithmic webs ; recent examples in some other veins. Abelian relations as the vanishing trace of particular 1-forms on the surface associated with $\mathcal{W}(d)$.

3. *From abelian relations to connection methods and applications*

Abelian relations as a local system through an effective "geometric" differential system. Linearization polynomial of a $\mathcal{W}(d)$ and its fundamental 1-form. Trends of ideas "à la Cartan-Spencer" : the associated connection (E, ∇) to $\mathcal{W}(d)$ and its curvature. Determinant connection $(\det E, \det \nabla)$. The connection for linear $\mathcal{L}(d)$ and the inverse Abel theorem. Two results from Ripoll's thesis : trace formula and rank determination.

4. *On singularities of planar webs and some open perspectives*

Singularities come on stage : the meromorphic connection (E, ∇) with poles on the discriminant Δ. Basic results on regular singular connections. Residues and the determinant formula for $(\det E, \det \nabla)$. At least three perspectives : Monodromy and regularity theorems for polynomial webs ; Towards a Galois theory for planar webs ; Poincaré-Blaschke methods *via* (E, ∇).

Abstract

Web geometry deals with families of foliations in general position. In these four lectures we restrict our attention to the planar case with complex analytic foliations of curves. Basic results and some new ones are presented essentially through nonlinear differential equations of the first order. We emphasize all along the talks on introductory examples : algebraic webs, polylogarithmic webs, effective methods in meromorphic connections, algebraic differential equations, *etc.* Moreover several open perspectives are also given on a subject which illustrates the rich interplay between differential geometry and algebraic geometry.