Abstract

Asymptotic expansions of heat kernel of Schrödinger-type operators, as well as its trace on noncompact spaces, are rarely explored, and even for cases as simple as $\mathbb{C}^n$ with (quasi-homogeneous) polynomials potentials, it's already very complicated to compute. Motivated by path integral formulation of the heat kernel, we introduced parabolic distance, which also appeared in Li-Yau's famous work on Harnack inequality. With the help of parabolic distance, we could derive a nice point-wise asymptotic expansion of the heat kernel with a strong remainder estimate. In particular, we get an asymptotic expansion of the heat kernel of Witten Laplacian $\Box_{Tf}$ induced by $d+Tdf\wedge$, where $T>0$ is the deformation parameter. When the deformation parameter of Witten deformation and time parameter are coupled, we derive an asymptotic expansion of trace of heat kernel for small-time t, and obtain a local index theorem. Also, we invented a new rescaling technique to write down the local index density explicitly. If time permits, I will also explain how we define Ray-Singer torsion on noncompact spaces. This is joint work with Xianzhe Dai.