



# 2025 分析、几何和方程研讨会


$$f_{xx} + f_{yy} = 0$$

南开大学陈省身数学研究所、数学科学学院

2025 年 12 月 11 日-14 日

天津

为了交流、报告当前关于分析、几何和方程相关领域的最新成果和进展，促进分析几何方程领域的应用和发展。南开大学将于 2025 年 12 月 11-14 日期间，在南开大学陈省身数学研究所举办 2025 年分析、几何和方程研讨会。

该研讨会由南开大学陈省身数学研究所提供经费和场地支持。

**会议时间安排：**

报到时间：12 月 11 日（下午 12:00-20:00）

学术报告时间：12 月 12 日-13 日全天

离会时间：12 月 14 日（全天）

报到地点：天津市卫津路 94 号南开大学嘉园宾馆

会议地点：南开大学陈省身研究所 216 教室

用餐地点：南开大学嘉园

**会议组织委员会：**

肖杰（纽芬兰纪念大学）

孙玉华（南开大学）

会议联系人：孙玉华 18522707104

**主办单位：**南开大学陈省身数学研究所

南开大学数学科学学院

12 月 12 日

地点: 省身楼 216

08:15 – 08:30 开幕式

08:30 – 09:10 主持人: 陈杰诚 (浙江师范大学) 报告人: 肖杰 (纽芬兰纪念大学)  
题目: Schrödinger Equations and Uncertainty Principles with Fractional Orders

09:10 – 09:50 主持人: 蒋庆堂 (浙江师范大学) 报告人: 谌稳固 (北京应用物理与计算数学研究所)  
题目: Casual Attention: Adaptive enforcement of causality in physics-informed neural networks

09:50 – 10:20 茶歇 & 合影

10:20 – 11:00 主持人: 李爱军 (浙江科技大学) 报告人: 周春琴 (上海交通大学)  
题目: Boundary pointwise regularity for the divergence form elliptic boundary problem on uniform domain

11:00 – 11:40 主持人: 刘聪文 (中国科学技术大学) 报告人: 李澎涛 (青岛大学)  
题目: Capacities related with parabolic Besov spaces

午餐 (嘉园)

14:00 – 14:40 主持人: 耿俊 (兰州大学) 报告人: 席东盟 (上海大学)  
题目: Minkowski problems and isoperimetric inequalities in affine convex geometry

14:40 – 15:20 主持人: 李俊峰 (大连理工大学) 报告人: 高晋 (杭州师范大学)  
题目: Maz'ya-Shaposhnikova type characterization for  $p$ -energies on metric measure spaces and fractals

15:20 – 15:40 茶歇

15:40 – 16:20 主持人: 刘丽光 (中国人民大学) 报告人: 韦勇 (中国科学技术大学)  
题目: Asymptotic Behaviour of the weak inverse anisotropic mean curvature flow

16:20 – 17:00 主持人: 郑神州 (北京交通大学) 报告人: 陈艳萍 (东北大学)  
题目: Some properties of generalized singular integrals

17:00 – 17:40 主持人: 卓然 (上海师范大学) 报告人: 洪寒 (北京交通大学)  
题目: Do Carmo's problem in Euclidean space and hyperbolic space

晚餐 (嘉园)

## 12 月 13 日

地点: 省身楼 216

**08:30 – 09:10**    主持人: 唐笑敏 (湖州师范学院)      报告人: 黄际政 (北京邮电大学)  
题目: Hardy-Sobolev spaces of higher order associated to Hermite operator

**09:10 – 09:50**    主持人: 王建飞 (华侨大学)      报告人: 吕小芬 (湖州师范学院)  
题目: Some work on tent spaces

**09:50 – 10:20**    茶歇

**10:20 – 11:00**    主持人: 刘宇 (北京科技大学)      报告人: 朱保成 (陕西师范大学)  
题目: The decomposition of geometric measures of convex body

**11:00 – 11:40**    主持人: 曹军 (浙江工业大学)      报告人: 张俊杰 (河北师范大学)  
题目: Regularity for nonlinear elliptic equations with measure data

午餐 (嘉园)

**14:00 – 14:40**    主持人: 苏华东 (北部湾大学)      报告人: 罗率兵 (湖南大学)  
题目: On Dirichlet-type and  $n$ -isometric shifts in finite rank de Branges-Rovnyak spaces

**14:40 – 15:20**    主持人: 靳晓尚 (华中科技大学)      报告人: 吴素青 (大连海事大学)  
题目: Riesz potentials for weighted mixed-norm Lebesgue spaces

**15:20 – 15:40**    茶歇

**15:40 – 16:20**    主持人: 蒋永生 (中南财经政法大学)      报告人: 张彤 (中国矿业大学)  
题目: Liouville-type theorem for the fractional  $p$ -Laplacian inequalities

**16:20 – 17:00**    主持人: 朱茂春 (南京理工大学)      报告人: 熊棋 (西南交通大学)  
题目: Riesz potential estimates for double obstacle problems with Orlicz growth

晚餐 (嘉园)

# ABSTRACTS

## Casual Attention: Adaptive enforcement of causality in physics-informed neural networks

谌稳固

北京应用物理与计算数学研究所

In this work, we propose an efficient, adaptive and gradient-less point-wise weighting method. The designed weights are non-negative, bounded, and monotonically decreasing in the temporal direction, thus imposing temporal causality in the training of PINNs. Moreover, the computation of the weights is completely decoupled from the spatiotemporal arrangement of collocation points, enabling seamless integration with various resampling algorithms. Extensive numerical examples demonstrate that the proposed method achieves state-of-the-art results. The weighting algorithm incorporated with resampling operations can effectively improve the convergence of PINNs in high-dimensional scenarios without succumbing to the curse of dimension.

## Some properties of generalized singular integrals

陈艳萍

东北大学

In this paper, we study the properties of generalized singular integrals and give some applications in SQG equation.

## Maz'ya-Shaposhnikova type characterization for p-energies on metric measure spaces and fractals

高晋

杭州师范大学

In this paper, we present the celebrated Bourgain-Brezis-Mironescu type characterization of p-energies in the setting of metric measure spaces, which is established via weak monotonic-

ity properties of various forms of  $p$ -energies. Then we extend the Maz'ya-Shaposhnikova type characterization, initially developed in Euclidean spaces, to the context of metric measure spaces. A notable aspect of our results is their validity in a broad class of spaces, including fractal sets. This work was carried out jointly with Zhenyu Yu and Junda Zhang.

## **Do Carmo's problem in Euclidean space and hyperbolic space**

洪寒

北京交通大学

Recent years have witnessed great progress on the stable Bernstein problem for minimal hypersurfaces in Euclidean space, particularly in lower dimensions. It has been completely solved up to ambient dimension 6. While the stable Bernstein problem was being studied, do Carmo raised a corresponding problem for constant mean curvature (CMC) hypersurfaces: must a complete, non-compact CMC hypersurface in Euclidean space with finite Morse index be minimal? The lower-dimensional cases ( $n \leq 5$ ) have been solved, but progress in higher dimensions has been limited. In this talk, I will discuss work joint with Jingche Chen and Haizhong Li in which we solved this problem in dimension 6. I will also mention some related progress on this problem in hyperbolic space.

## **Hardy-Sobolev spaces of higher order associated to Hermite operator**

黄际政

北京邮电大学

In this talk, we will consider the Hardy-Sobolev spaces of higher order associated with Hermite operator. We also give some new characterizations of the Hardy spaces associated with Hermite operator.

# Capacities related with parabolic Besov spaces

李澎涛

青岛大学

This paper investigates geometric characterizations of Carleson-type embeddings of parabolic Besov-type spaces via a parabolic degenerate extension problem associated with the fractional heat operator.

# On Dirichlet-type and $n$ -isometric shifts in finite rank de Branges-Rovnyak spaces

罗率兵

湖南大学

We study the function spaces  $\mathcal{D}(\mu)$  by Richter and Aleman, and  $\mathcal{D}_{\vec{\mu}}$  by Rydhe. It is known that the forward shift  $M_z$  is bounded and expansive on  $\mathcal{D}(\mu)$ , and therefore  $\mathcal{D}(\mu)$  coincides with a de Branges-Rovnyak space  $\mathcal{H}(B)$ . We show that such a  $B$  is rational if and only if  $\mu$  is finitely atomic, and this happens exactly when the corresponding defect operator has finite rank. We also outline a method for calculating the reproducing kernel of  $\mathcal{D}(\mu)$  for finitely atomic  $\mu$ . Similarly, we characterize the allowable tuples  $\vec{\mu} = (\frac{|dz|}{2\pi}, \mu_1, \dots, \mu_{n-1})$  such that  $M_z$  on  $\mathcal{D}_{\vec{\mu}}$  is expansive with finite rank defect operator. Our investigation provides many interesting examples of normalized allowable tuples  $\vec{\mu}$ .

# Some work on tent spaces

吕小芬

湖州师范学院

In this talk, we introduce the definitions of tent space and tent-Carleson measure, and characterize the positive Borel measure such that it is a (vanishing) tent-Carleson measures for some holomorphic function spaces, including Hardy space, Bergman space,  $Q_p$  space.

# Asymptotic Behaviour of the weak inverse anisotropic mean curvature flow

韦勇

中国科学技术大学

We first establish a local gradient estimate for anisotropic  $p$ -harmonic functions. A key feature of our estimate is that the constant remains bounded as  $p \rightarrow 1$ ; consequently, in the limit  $p \rightarrow 1$ , this estimate yields the local gradient estimate for weak solutions of the inverse anisotropic mean curvature flow (IAMCF). As an application, we show that the weak IAMCF is asymptotic to the expanding Wulff shape solution at the infinity, thereby extending the result of Huisken and Ilmanen (2001) to the anisotropic case. Joint with Chaoqun Gao and Rong Zhou.

# Riesz potentials for weighted mixed-norm Lebesgue spaces

吴素青

大连海事大学

In this talk, we will introduce the Riesz capacity associated with the weighted mixed-norm Lebesgue spaces, and establish the corresponding capacity inequalities. The approach taken is mainly based on a new characterization of weighted mixed-norm Lebesgue spaces via using an operator  $T_\beta^b$ , where  $b \geq 1$  is close to 1. This operator is defined by using either the Taylor remainder (when  $\beta$  is a non-integer) or the high order difference (when  $\beta$  is an integer) of the kernel of  $\mathcal{I}_\beta^{(2n)}$ . This is a joint work with Dalian Jin, Liguang Liu and Jie Xiao.

# Minkowski problems and isoperimetric inequalities in affine convex geometry

席东盟

上海大学

While affine isoperimetric inequalities compare affine geometric invariants (such as volumes and affine surface areas), affine geometric measures arise from the variations of these

affine invariants. We study the associate prescribe measure problem (affine Minkowski problems) and related affine inequalities.

## **Schrödinger Equations and Uncertainty Principles with Fractional Orders**

肖杰

纽芬兰纪念大学

This talk will address the fractional Schrödinger equations and their associated Uncertainty principles.

## **Riesz potential estimates for double obstacle problems with Orlicz growth**

熊棋

西南交通大学

In this paper, we consider the solutions to the non-homogeneous double obstacle problems with Orlicz growth involving measure data. After establishing the existence of the solutions to this problem in the Orlicz-Sobolev space, we derive a pointwise gradient estimate for these solutions by Riesz potential, which leads to the result on the  $C^1$  regularity criterion.

## **Regularity for nonlinear elliptic equations with measure data**

张俊杰

河北师范大学

In this talk, we consider general nonlinear elliptic equations of the form  $-\operatorname{div} \mathcal{A}(x, u, Du) = \mu$ , whose principle part  $\mathcal{A}$  depends on the solution itself and right-hand data  $\mu$  is a signed Radon measure. The associated nonlinearity  $\mathcal{A}$  is assumed to satisfy the small BMO condition in  $x$ , Lipschitz continuity condition in  $u$ , and  $p$ -growth, Orlicz growth or  $p(x)$ -growth condition in  $Du$ , while the boundary of underlying domain is assumed to be Reifenberg flat. We will establish an optimal global Calderón-Zygmund type estimate for gradients of very

weak solutions to such measure data problem.

## **Liouville-type theorem for the fractional $p$ -Laplacian inequalities**

张彤

中国矿业大学

This work addresses an open question posed in [Math. Ann. 2022, "Quasilinear Laplace equations and inequalities with fractional orders"]. In the process of resolving this problem, we uncovered a key structural insight. This discovery enables us to establish the existence of solutions for a broad class of fractional  $p$ -Laplacian inequalities, extending to cases with logarithmic, exponential, and power-law decay at infinity. (Joint work with Professor Jie Xiao.)

## **Boundary pointwise regularity for the divergence form elliptic boundary problem on uniform domain**

周春琴

上海交通大学

In this paper, we study the boundary pointwise regularity for the divergence form elliptic boundary problem. In generality, it is not convenient to define weak solutions for nonzero boundary data on domain with the rough boundary, e.g. uniform domain. However, in this paper, we introduce a definition of weak solutions for the boundary problem on uniform domain. What is interesting is that this definition can be considered to analysis the regularity of weak solutions. In particular, by establishing the energy inequality, we show the boundary pointwise  $C^\alpha$  regularity by using compactness methods under the admissible condition. Furthermore, by establishing the linear property of solutions with respect to the harmonic functions, we also prove the boundary pointwise  $C^{1,\alpha}$  and  $C^{2,\alpha}$  regularities if the boundary data and the boundary of domain are pointwise  $C^{1,\alpha}$  and  $C^{2,\alpha}$  respectively.

# The decomposition of geometric measures of convex body

朱保成

陕西师范大学

We will talk about the decomposition formulas for the curvature measures of convex bodies. The decomposition of a curvature measure of a convex body is with respect to Hausdorff measures of different dimensions restricted to the singular sets of the boundary of the convex body. The density functions and singular measures in the decomposition are explicitly given in terms of integrals of functions of the generalized curvatures of the convex body.

序号	参会人员	单位
1	肖杰	Memorial University
2	曹军	浙江工业大学
3	常迪闻	北京工商大学
4	陈奥博	清华大学
5	陈杰诚	浙江师范大学
6	陈艳萍	东北大学
7	谌稳固	北京应用物理与计算数学研究所
8	董兴堂	天津大学
9	房成龙	福州大学
10	付星	湖北大学
11	高晋	杭州师范大学
12	耿俊	兰州大学
13	顾庆松	南京大学
14	洪寒	北京交通大学
15	胡二彦	天津大学
16	胡海丰	长春大学
17	虎文婷	河南师范大学
18	黄际政	北京邮电大学
19	黄曼子	湖南师范大学
20	贾荔茜	潍坊学院
21	蒋庆堂	浙江师范大学
22	蒋永生	中南财经政法大学
23	靳晓尚	华中科技大学
24	李爱军	浙江科技大学
25	李俊峰	大连理工大学
26	李康伟	天津大学
27	李澎湃	青岛大学
28	刘聪文	中国科学技术大学
29	刘丽光	中国人民大学
30	刘宇	北京科技大学
31	罗率兵	湖南大学
32	吕小芬	湖州师范学院
33	石少广	临沂大学
34	苏华东	北部湾大学
35	唐笑敏	湖州师范学院
36	万海涛	山东工商学院
37	王建飞	华侨大学
38	王壮	湖南师范大学
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42	熊棋	西南交通大学
43	徐帆恒	中山大学
44	许栩	中国地质大学（北京）
45	余振宇	国防科技大学
46	袁程	广东工业大学
47	张俊杰	河北师范大学
48	张骏达	华南理工大学
49	张宁	华中科技大学
50	张彤	中国矿业大学
51	赵国焕	中国科学院数学与系统科学 研究院
52	郑神州	北京交通大学
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56	朱茂春	南京理工大学
57	卓然	上海师范大学
校内参会名单		
1	陈婷	南开大学
2	曹杰	南开大学
2	李磊	南开大学
3	吕福生	南开大学
4	高泳昕	南开大学
5	马羚未	南开大学
6	马世光	南开大学
7	孙文昌	南开大学
8	孙玉华	南开大学
9	王文龙	南开大学
10	王险峰	南开大学
11	张震球	南开大学