Abstract

We present a rigorous derivation of the continuum Kimura equation from a discrete Wright-Fisher genetic drift model. We show that boundary conditions are not needed for and cannot be imposed on the resulting degenerate diffusion problem. To this end, we reformulate the concept of weak solutions. In doing so, we find that the extension of the Kimura equation to the whole space should be the continuum limit that carries over the biologically relevant statistic information from the discrete model; namely, the conservation laws embedded in the discrete model are now selfcontained in the continuum problem, without imposing any extra boundary conditions or integral constraints. We establish a well-posedness (existence, uniqueness, regularity, stability) theory and especially prove the analyticity of the solution. Our arguments build an intrinsic connection between the genetic fixation probability and a stochastic process with two absorbing barriers.