Introduction

Chemotaxis, the partially oriented motion of individuals in response to concentration gradients of a chemical substance, is known as a ubiquitously observable mechanism of interaction which plays outstanding roles in numerous processes of self-organization at all levels of complexity of biological systems. According to various modeling approaches, an adequate mathematical description thereof requires the study of parabolic systems of partial differential equations involving certain cross-diffusive terms as their most characteristic ingredient.

This series of lectures attempts to describe some analytical approaches toward an understanding of how such mechanisms may influence the qualitative behavior of solutions. A particular focus will be on the development of methods capable of detecting phenomena of spontaneous aggregation in the mathematically extreme flavor of finite-time blow-up.

Plan of the course:

1.Local existence and extensibility

2.Approaches toward global solvability. Boundedness in various

system classes

3.Finite-time blow-up in the fully parabolic Keller-Segel system

4.Discovering explosions in more complex systems. Moment-based strategies

5.Global solution theories relying on variational approaches