Abstract

Let M be a closed symplectic manifold of dimension 2n with non-ellipticity.

We can dene an almost K aehler structure on M by using the given symplectic form. Using Darboux coordinate charts, we deform the given almost K aehler structure to obtain a homotopy equivalent Lipschitz K aehler structure on the universal covering of M. Analogous to Teleman's L^2-Hodge decomposition on PL manifolds or Lipschitz Riemannian manifolds, we give a L^2-Hodge decomposition theorem on the universal covering of M w.r.t. the measurable K aehler metric. Using an argument of Gromov, we give a vanishing theorem for L^2 harmonic p-forms, where p is not equal to n (resp. a non-vanishing theorem for L^2 harmonic n-forms) on the universal covering of M, then its signed Euler characteristic satises (-1)^n\chi(M) 0 (resp. (-1)^n\chi(M) > 0). As an application, we show that the Chern-Hopf conjecture holds true in closed even dimensional Riemannian manifolds with nonpositive curvature (resp. strictly negative curvature), it gives a positive answer to a Yau's problem due to S. S. Chern and H. Hopf.