

Abstract

In mathematics, the main goal is not just to prove theorems and solve problems, but to discover the right problems and the right questions in the first place. This sounds like an obvious remark, but in fact it is often not at all clear when one first thinks of a new mathematical question whether it is a "good" one, meaning one that will be fruitful in leading to new discoveries and mathematical insights, or a "bad" one, meaning that it may sound appealing at first sight but in fact does not lead anywhere. I will start the lecture by giving two examples of nice mathematical statements, both of which are true but one of which turns out to have deep connections with many parts of mathematics, while the other is (as far as we know) just an amusement, and will ask the members of the audience to vote on which one they think is which. The main part of the talk will give many other examples drawn from number theory and more specifically from the theory of Diophantine equations, named after the 2nd century Greek mathematician Diophantus. Here, too, many quite natural-sounding questions turn out to be dead ends, but others lead to an incredibly fertile subject (called the theory of elliptic curves). I will talk about this class of problems in a very general way in the colloquium talk, and will

then go into the subject in more detail in my Math Center seminar talk on December 16, but still at an elementary level that is meant to be accessible to bright undergraduates and to be of interest to mathematicians from all fields.