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On the asymptotic expansion of Bergman kernel

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Abstract

We study the asymptotics of the Bergman kernel and the heat kernel of the spin^c Dirac operator on high tensor powers of a line bundle. ***To cite this article: X. Dai et al., C. R. Acad. Sci. Paris, Ser. I 339 (2004).***

Résumé

Sur le développement asymptotique du noyau de Bergman. On étudions les développements asymptotiques du noyau de la chaleur et de Bergman de l'opérateur de Dirac spin^c associé à une puissance grande d'un fibré en droites positif. ***Pour citer cet article : X. Dai et al., C. R. Acad. Sci. Paris, Ser. I 339 (2004).***

Version française abrégée

Le noyau de Bergman des variétés projectives a été étudié en particulier dans [13,12,15,5,9], où on a établi le développement asymptotique du noyau associé à une puissance tendant vers $+\infty$ d'un fibré en droites positif. Les coefficients de ce développement donnent des informations géométriques sur la variété projective associée. Ce développement asymptotique joue un rôle crucial dans un travail récent de Donaldson [7], où l'existence d'une métrique Kählérienne de courbure scalaire constante est reliée à la stabilité de Mumford–Chow.

Dans cette Note, on étudions le développement asymptotique de ce noyau dans le cas plus général des variétés symplectiques ou orbifolds symplectiques. On étudie aussi le développement asymptotique du noyau de la chaleur correspondant, et on le relie au développement du noyau de Bergman. Une autre motivation de ce travail est d'étendre le travail récent de Donaldson au cas des orbifolds.

Les résultats annoncés dans cette note sont démontrés dans [6].

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1. Introduction

The Bergman kernel in the context of several complex variables (i.e. for pseudoconvex domains) has long been an important subject. Its analogue for compact complex projective manifolds is studied in [13,12,15,5,9], where its asymptotic expansion for high powers of an ample line bundle is established. Moreover, the coefficients in the asymptotic expansion encode geometric information of the underlying complex projective manifolds. This asymptotic expansion plays a crucial role in the recent work of [7] where the existence of Kähler metrics with constant scalar curvature is shown to be closely related to the Mumford–Chow stability.

In this Note, we study the asymptotic expansion of Bergman kernel for high powers of an ample line bundle in the more general context of symplectic manifolds and orbifolds. We also study the asymptotic expansion of the corresponding heat kernel and relates it to that of the Bergman kernel. One of our motivations is to extend Donaldson's recent work to orbifolds.

The full details of our results are given in [6].

2. Bergman kernels and heat kernels

Let (X,ω) be a compact symplectic manifold of real dimension $2n$. Assume that there exists an Hermitian line bundle L over X endowed with an Hermitian connection ∇^L with the property that $\frac{\sqrt{-1}}{2\pi}R^L=\omega$, where $R^L=(\nabla^L)^2$ is the curvature of (L,∇^L). Let (E,h^E) be an Hermitian vector bundle on X with Hermitian connection ∇^E and curvature R^E.

Let g^{TX} be a Riemannian metric on X. Let $\mathbf{J}:TX\to TX$ be the skew-adjoint linear map which satisfies the relation

$$\omega(u,v)=g^{TX}(\mathbf{J}u,v) \tag{1}$$

for $u,v\in TX$. Let J be an almost complex structure which is (separately) compatible with g^{TX} and ω, especially, $\omega(\cdot,J\cdot)$ defines another metric on TX; then J commutes with $\mathbf{J}$. The almost complex structure J induces a splitting $T_{\mathbb{R}}X\otimes_{\mathbb{R}}\mathbb{C}=T^{(1,0)}X\oplus T^{(0,1)}X$, where $T^{(1,0)}X$ and $T^{(0,1)}X$ are the eigenbundles of J corresponding to the eigenvalues $\sqrt{-1}$ and $-\sqrt{-1}$ respectively. Let $T^{*(1,0)}X$ and $T^{*(0,1)}X$ be the corresponding dual bundles.

Let $\mathrm{d}v_X$ be the Riemannian volume form of (TX,g^{TX}). The above data induce naturally a scalar product $\langle s_1,s_2\rangle=\int_X\langle s_1(x),s_2(x)\rangle_{\Lambda^{0,\bullet}\otimes L^k\otimes E}\,\mathrm{d}v_X(x)$, on $\Omega^{0,\bullet}(X,L^p\otimes E)=\bigoplus_{q=0}^n\Omega^{0,q}(X,L^p\otimes E)$, the direct sum of spaces of $(0,q)$-forms with values in $L^p\otimes E$.

Let ∇^{TX} be the Levi-Civita connection on (TX,g^{TX}) with curvature R^{TX}. We denote by $P^{T^{(1,0)}X}$ the projection from $T_{\mathbb{R}}X\otimes_{\mathbb{R}}\mathbb{C}$ to $T^{(1,0)}X$. Let $\nabla^{T^{(1,0)}X}=P^{T^{(1,0)}X}\nabla^{TX}P^{T^{(1,0)}X}$ be the Hermitian connection on $T^{(1,0)}X$ induced by ∇^{TX} with curvature $R^{T^{(1,0)}X}$. Let $\nabla^{\det(T^{(1,0)}X)}$ be the connection on $\det(T^{(1,0)}X)$ induced by $\nabla^{T^{(1,0)}X}$ with curvature $R^{\det}=\mathrm{Tr}[R^{T^{(1,0)}X}]$. Then spin^c Dirac operator D_p acts on $\Omega^{0,\bullet}(X,L^p\otimes E)$ (cf. also [11, §2]).

Let P_p be the orthogonal projection from $\Omega^{0,\bullet}(X,L^p\otimes E)$ on $\operatorname{Ker}D_p$. Let $P_p(x,x')$, $\exp(-\frac{u}{p}D_p^2)(x,x')$ be the smooth kernel of P_p, and of the heat kernel $\exp(-\frac{u}{p}D_p^2)$ with respect to $dv_X(x')$. Especially, $P_p(x,x)$, $\exp(-\frac{u}{p}D_p^2)(x,x)\in\mathrm{End}(\Lambda(T^{*(0,1)}X)\otimes E)_x$.

We denote by $I_{\mathbb{C}\otimes E}$ the projection from $\Lambda(T^{*(0,1)}X)\otimes E$ onto $\mathbb{C}\otimes E$ under the decomposition $\Lambda(T^{*(0,1)}X)=\mathbb{C}\oplus\Lambda^{>0}(T^{*(0,1)}X)$. Let $\det\mathbf{J}$ be the determinant function of $\mathbf{J}_x\in\mathrm{End}(T_xX)$. One of our main results is

Theorem 2.1. *There exist smooth coefficients $b_r(x)\in\mathrm{End}(\Lambda(T^{*(0,1)}X)\otimes E)_x$ which are polynomials in R^{TX}, $R^{\det}$, R^E (and R^L) and their derivatives with order $\leqslant 2r-1$ (resp. $2r$) and $\mathbf{J}^{-1}$ at x, and $b_0=(\det\mathbf{J})^{1/2}I_{\mathbb{C}\otimes E}$, such that for any $k,l\in\mathbb{N}$, there exists $C_{k,l}>0$ such that for any $x\in X$, $p\in\mathbb{N}$,*

$$\left|P_p(x,x)-\sum_{r=0}^{k} b_r(x)p^{n-r}\right|_{C^l} \leqslant C_{k,l}\,p^{n-k-1}. \tag{2}$$

Moreover, the expansion is uniform in the sense that for any $k,l\in\mathbb{N}$, there is an integer s such that if all data $(g^{TX},\ h^L,\ \nabla^L,\ h^E,\ \nabla^E,\ J)$ run over a set which are bounded in C^s and with g^{TX} bounded below, then the constants $C_{k,l}$ are independent of g^{TX}.

We also have the following large p asymptotic expansion for the heat kernel.

Theorem 2.2. *There exist smooth sections $b_{r,u}$ of $\mathrm{End}(\Lambda(T^{*(0,1)}X)\otimes E)$ on X such that for each $u>0$ fixed, we have the asymptotic expansion in the sense of (2) as $p\to\infty$,*

$$\exp\left(-\frac{u}{p}D_p^2\right)(x,x)=\sum_{r=0}^{k} b_{r,u}(x)p^{n-r}+\mathcal{O}\left(p^{n-k-1}\right). \tag{3}$$

Moreover, there exists $c>0$ such that as $u\to+\infty$,

$$b_{r,u}(x)=b_r(x)+\mathcal{O}\left(\mathrm{e}^{-cu}\right). \tag{4}$$

In fact, this gives us a way to compute the coefficient $b_r(x)$, as it is relatively easy to compute $b_{r,u}(x)$. As an example, we compute b_1 which plays an important role in Donaldson's recent work [7]. Note if (X,ω) is Kähler and $\mathbf{J}=J$, then $B_p(x)\in C^\infty(X,\mathrm{End}(E))$ for p big enough, thus $b_r(x)\in\mathrm{End}(E)_x$.

Theorem 2.3. *If (X,ω) is Kähler and $\mathbf{J}=J$, then there exist smooth functions $b_j(x)\in\mathrm{End}(E)_x$ such that we have (2), and b_j are polynomials in R^{TX}, R^E and their derivatives with order $\leqslant 2r-1$ at x, and*

$$b_0=\mathrm{Id}_E,\quad b_1=\frac{1}{4\pi}\left[\sqrt{-1}\sum_i R^E(e_i,Je_i)+\frac{1}{2}r^X\,\mathrm{Id}_E\right], \tag{5}$$

here r^X is the scalar curvature of (X,g^{TX}), and $\{e_i\}$ is an orthonormal basis of (X,g^{TX}).

Theorem 2.3 was essentially obtained in [9,14] by applying the peak section trick, and in [5] and [15] by applying the Boutet de Monvel–Sjöstrand parametrix for the Szegö kernel [4]. We refer the reader to [7,14] for interesting applications. Our proof of Theorems 2.1, 2.2 is inspired by local Index Theory, especially from [2, § 11]. It can be easily generalized to the orbifold situation.

Let (X,ω) be a compact symplectic orbifold of real dimension $2n$ with singular set X' (cf. [8]). By definition, for any $x\in X$, there exists a small neighborhood $U_x\subset X$, a finite group G_x acting linearly on $\mathbb{R}^{2n}$, and $\widetilde{U}_x\subset\mathbb{R}^{2n}$ an G_x-open set such that $\widetilde{U}_x\xrightarrow{\tau_x}\widetilde{U}_x/G_x=U_x$ and $\{0\}=\tau_x^{-1}(x)\in\widetilde{U}_x$.

An orbifold vector bundle E on an orbifold X is such that for any $x\in X$, there exists $\tilde{p}_{U_x}:\widetilde{E}_{U_x}\to\widetilde{U_x}$ a $G^E_{U_x}$-equivariant vector bundle and $(G^E_{U_x},\widetilde{E}_{U_x})$ (resp. $(G^E_{U_x}/K_{U_x},\widetilde{U}_x)$, $K_{U_x}=\mathrm{Ker}(G^E_{U_x}\to\mathrm{Diffeo}(\widetilde{U}_x))$) is the orbifold structure of E (resp. X). We say E is proper if $G^E_{U_x}=G_x$ for any $x\in X$. For any orbifold vector bundle E, its proper part is a proper orbifold vector bundle.

Now, any structure on X or E should be locally G_x or $G^E_{U_x}$ equivariant.

Theorem 2.4. *If (X,ω) is a symplectic orbifold with singular set X', and L, E are corresponding proper orbifold vector bundles on X as in Theorem 2.1. Then there exist smooth coefficients $b_r(x)\in\mathrm{End}(\Lambda(T^{*(0,1)}X)\otimes E)_x$ with $b_0=(\det\mathbf{J})^{1/2}I_{\mathbb{C}\otimes E}$, and $b_r(x)$ which are polynomials in R^{TX}, $R^{\det}$, R^E (and R^L) and their derivatives with* *order* $\leqslant 2r-1$ (*resp.* $2r$) *and* $\mathbf{J}^{-1}$ *at* x*, such that for any* $k,l\in\mathbb{N}$*, there exist* $C_{k,l}>0,\ N\in\mathbb{N}$ *such that for any* $x\in X,\ p\in\mathbb{N}$,

$$\left|\frac{1}{p^n}P_p(x,x)-\sum_{r=0}^{k}b_r(x)p^{-r}\right|_{C^l}\leqslant C_{k,l}\big(p^{-k-1}+p^{l/2}\big(1+\sqrt{p}d(x,X')\big)^N \mathrm{e}^{-C\sqrt{p}d(x,X')}\big). \tag{6}$$

Moreover if the orbifold (X,ω) *is Kähler,* $\mathbf{J}=J$ *and the proper orbifold vector bundles* E,L *are holomorphic on* X*, then* $b_r(x)\in\mathrm{End}(E)_x$ *and* $b_r(x)$ *are polynomials in* R^{TX}*,* R^E *and their derivatives with order* $\leqslant 2r-1$ *at* x*.*

3. Idea of the proofs

Our first observation is that by [11, Theorem 0.1], [3, Theorem 1] exist $\mu_0, C_L>0$ such that

$$\operatorname{Spec} D_p^2\subset\{0\}\cup[2p\mu_0-C_L,+\infty[. \tag{7}$$

Now by (7), using finite propagation speed for solutions of hyperbolic equation, we can localize the problem. In particular, the asymptotics of $P_p(x_0,x')$ as $p\to\infty$ is localized on a neighborhood of x_0.

For $x_0\in X,\ \varepsilon>0$, let $B^{T_{x_0}X}(0,\varepsilon)$ be the open ball in $T_{x_0}X$ with center x_0 and radius ε, we identify it with a neighborhood of $x_0\in X$ by using the exponential map. We also identify $(L,h^L),\ (E,h^E)$ with $(L_{x_0},h^{L_{x_0}})$, $(E_{x_0},h^{E_{x_0}})$ respectively on a neighborhood of 0 by using the parallel transport with respect to ∇^L, ∇^E along the radial direction.

We replace the manifold X by $\mathbb{R}^{2n}\simeq T_{x_0}X=X_0$, and we extend the bundles and connections to the full $T_{x_0}X$. In particular, we can extend ∇^L (resp. ∇^E) to a Hermitian connection ∇^{L_0} on $(L_{x_0},h^{L_{x_0}})$ (resp. ∇^{E_0} on $(E_{x_0},h^{E_{x_0}})$) on $T_{x_0}X$ in such a way so that we still have positive curvature R^{L_0}; in addition $R^{L_0}=R^L_{x_0}$ outside a compact set. Also the metric g^{TX_0}, the almost complex structure J_0, (resp. the connection ∇^{E_0}) are extended in such a way that they coincide with the corresponding ones at 0 (resp. the trivial connection) outside a compact set. Now, we fix a unit vector $S_L\in L_{x_0}$, then using S_L and the above discussion, we can get an isometry $\Lambda(T^{*(0,1)}X_0)\otimes L_0^p\otimes E_0\simeq(\Lambda(T^{*(0,1)}X)\otimes E)_{x_0}=\mathbf{E}_{x_0}$.

Let $D_p^{X_0}$ be the Dirac operator on X_0 associated to the above data. Then (7) still holds for $D_p^{X_0}$. Let P_p^0 be the orthogonal projection from $\Omega^{0,\bullet}(X_0,L_0^p\otimes E_0)\simeq C^\infty(X_0,\mathbf{E}_{x_0})$ on $\operatorname{Ker}D_p^{X_0}$, and let $P_p^0(x,x')$ be the smooth kernel of P_p^0 with respect to the volume form $\mathrm{d}v_{X_0}(x')$. Let $\mathrm{d}v_{TX}$ be the Riemannian volume form on $(T_{x_0}X,g^{T_{x_0}X})$. Let $\kappa(Z)$ be the smooth positive function defined by the equation

$$\mathrm{d}v_{X_0}(Z)=\kappa(Z)\,\mathrm{d}v_{TX}(Z), \tag{8}$$

with $k(0)=1$. For $s\in C^\infty(\mathbb{R}^{2n},\mathbf{E}_{x_0})$ and $Z\in\mathbb{R}^{2n}$, for $t=1/\sqrt{p}$, set

$$(S_ts)(Z)=s(Z/t),\qquad L_2^t=S_t^{-1}t^2D_p^{X_0,2}S_t. \tag{9}$$

For $s\in C^\infty(T_{x_0}X,\mathbf{E}_{x_0})$, set

$$\|s\|_{t,0}^2=\int_{\mathbb{R}^{2n}}\big|s(Z)\big|^2_{h^{\Lambda(T^{*(0,1)}X_0)\otimes E_0}(tZ)}\,\mathrm{d}v_{X_0}(tZ). \tag{10}$$

Then L_2^t is a formally self-adjoint elliptic operator with respect to $\|\cdot\|_{t,0}^2$, and is a smooth family of differential operators with parameter $x_0\in X$ and coefficients in $\mathrm{End}(\mathbf{E}_{x_0})=\mathrm{End}(\Lambda(T^{*(0,1)}X)\otimes E)_{x_0}$. Let $\pi:TX\times_X TX\to X$ be the natural projection from the fiberwise product of TX on X.

Let $P_{0,t}$ be the orthogonal projection from $C^\infty(X_0,\mathbf{E}_{x_0})$ to the kernel of L_2^t with respect to $\|\cdot\|_{t,0}$. Set

$$F_u\big(L_2^t\big)=\mathrm{e}^{-uL_2^t}-P_{0,t}=\int_u^{+\infty} L_2^t\,\mathrm{e}^{-u_1L_2^t}\,\mathrm{d}u_1. \tag{11}$$

Let $P_{0,t}(Z,Z')$, $\mathrm{e}^{-uL_2^t}(Z,Z')$, $F_u(L_2^t)(Z,Z')$ be the smooth kernels of the operators $P_{0,t}$, $\mathrm{e}^{-uL_2^t}$, $F_u(L_2^t)$ with respect to $\mathrm{d}v_{TX}(Z')$. Then we can view these kernels as smooth sections of $\pi^*(\mathrm{End}(\Lambda(T^{*(0,1)}X)\otimes E))$ on $TX\times_X TX$. In (12), $|\cdot|_{C^m(X)}$ is the C^m-norm for the parameter $x_0\in X$.

From (7), (11), we can get the following key estimate by introducing a family of Sobolev norm on $C^\infty(X_0,\mathbf{E}_{x_0})$, and by extending the functional analysis techniques in [2, §11],

Theorem 3.1. *There exists $C''>0$ such that for any $k,m,m'\in\mathbb{N}$, $u_0>0$, there exist $N\in\mathbb{N}$, $C>0$ such that if $t\in\,]0,1]$, $u\geqslant u_0$, $Z,Z'\in T_{x_0}X$,*

$$\begin{aligned}
&\sup_{|\alpha|,|\alpha'|\leqslant m}\left|\frac{\partial^{|\alpha|+|\alpha'|}}{\partial Z^\alpha\partial Z'^{\alpha'}}\left(F_u\big(L_2^t\big)-\sum_{r=0}^k F_{r,u}t^r\right)(Z,Z')\right|_{C^{m'}(X)}\\
&\quad\leqslant Ct^{k+1}\big(1+|Z|+|Z'|\big)^N\exp\left(-\frac{1}{8}\mu_0u-\sqrt{C''\mu_0}|Z-Z'|\right),\\
&\sup_{|\alpha|,|\alpha'|\leqslant m}\left|\frac{\partial^{|\alpha|+|\alpha'|}}{\partial Z^\alpha\partial Z'^{\alpha'}}\left(\mathrm{e}^{-uL_2^t}-\sum_{r=0}^k J_{r,u}t^r\right)(Z,Z')\right|_{C^{m'}(X)}\\
&\quad\leqslant Ct^{k+1}\big(1+|Z|+|Z'|\big)^N\exp\left(\frac{1}{2}\mu_0u-\frac{2C''}{u}|Z-Z'|^2\right).
\end{aligned}\tag{12}$$

Now there are second order differential operators $\mathcal{Q}_r$ whose coefficients are polynomials in Z with coefficients polynomials in R^{TX}, $R^{\det}$, R^E, R^L and their derivatives at x_0, such that

$$L_t^2=L_2^0+\sum_{r=1}^\infty \mathcal{Q}_r t^r. \tag{13}$$

We obtain the coefficients $J_{r,u}$ from the Volterra expansion of $\mathrm{e}^{-uL_2^t}$ (cf. [1, §2.4]).

By (9), for $Z,Z'\in T_{x_0}X$,

$$\begin{aligned}
&P_p^0(Z,Z')=p^nP_{0,t}(Z/t,Z'/t)\kappa^{-1}(Z'),\\
&\exp\left(-\frac{u}{p}D_p^{X_0,2}\right)(Z,Z')=p^n\,\mathrm{e}^{-uL_2^t}(Z/t,Z'/t)\kappa^{-1}(Z').
\end{aligned}\tag{14}$$

From (11), Theorem 3.1, with $Z,Z'=0$, we deduce (6), and $b_r(x_0)=J_{r,u}(0,0)-F_{r,u}(0,0)$. From Theorem 3.1, we also know that $F_{r,u}$ is estimated by the coefficient of t^{k+1} at the right-hand side of the first equation of (12). In particular, $F_{r,u}(0,0)=\mathrm{O}(\mathrm{e}^{-\frac{1}{8}\mu_0u})$ as $u\to\infty$. This completes the proof of Theorems 2.1 and 2.2.

If (X,ω) is Kähler and $\mathbf{J}=J$, then $\mathcal{Q}_1=0$ and

$$J_{2,u}=-\int_0^u \mathrm{e}^{-(u-u_1)L_2^0}\mathcal{Q}_2\,\mathrm{e}^{-u_1L_2^0}\,\mathrm{d}u_1. \tag{15}$$

Now $b_1(x_0)=\lim_u J_{2,u}(0,0)$. Thus we derive (5). We also establish Theorem 2.4 from Theorem 3.1 by using finite propagation speed on orbifolds as in [10].

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