【课程名称】Course

最优运输与蒙日-安培方程简介 Introduction to Optimal Transportation and Monge-Ampere Equation

【先修课程】Prerequisite knowledge

数学分析，线性代数，泛函分析，偏微分方程
Calculus, Linear Algebra, Functional Analysis, PDE

【授课方式】Teaching method

以课堂讲授为主，辅以课堂讨论
Lectures and discussions

【课程简介】Course Description

Chinese version: 本课程旨在简要介绍最优运输问题以及它在其他领域中的应用。课程内容包括最优运输映射的存在唯一性和正则性，蒙日-安培方程的正则性理论。

English version: This subject gives an introduction to the optimal transportation problem, which arises in a broad range of areas: Fluid Mechanics; Partial Differential Equations (PDE); Optimisation; and Financial Mathematics. From an analytic point of view, this subject introduces the elementary existence and uniqueness theory, with a focus on recent development on regularity theory. It involves the study of the Monge-Ampère type PDE, whose applications extend to more areas, in particular, in calculus of variations and geometry.

【课程大纲】Course syllabus

第 0 章：内容简介
第 1 章：Kantorovich 对偶泛函
第 2 章：最优运输的几何性质
第 3 章：Brenier 的极化分解定理
第 4 章：Monge-Ampère 方程
第 5 章：严格凸与 C1 正则性
第 6 章：自然边值条件与 C1,alpha 正则性
第 7 章：C2,alpha 与高阶正则性
第 8 章：整体正则性
Chapter 0: Introduction
Chapter 1: The Kantorovich Duality
Chapter 2: Geometry of Optimal Transportation
Chapter 3: Brenier's Polar Factorization Theorem
Chapter 4: The Monge-Ampère Equation
Chapter 5: Strict convexity and C1 regularity
Chapter 6: Natural boundary condition and C1,alpha regularity
Chapter 7: C2,alpha and higher order regularities
Chapter 8: Global regularity

【参考读物】 Reference material

Cedric Villani. 2003. Topics in Optimal Transportation. *Graduate Studies in Mathematics* 58, American Mathematical Society, Providence, Rhode Island.

Villani, C., 2009. Optimal Transportation, Old and New, *Grundlehren der Math. Wiss. 338. Springer-Verlag, Berlin*

Figalli, A., 2017. The Monge-Ampère equation and its applications, *Zurich Lectures in Advanced Mathematics*. European Mathematical Society (EMS).

Gilbarg, D. and Trudinger, N. S., 1983. Elliptic partial differential equations of second order. *Springer-Verlag, Berlin*.