Abstract

In this talk, we first present some new sharp Li–Yau type gradient estimates for the positive solution $u(x,t)$ of the heat equations

$$(\partial t-\Delta)u=0$$

on a complete manifold with $Ric(M) \geq - k$ for $k \geq 0$. As application, new parabolic Harnack inequalities are derived, Base on our new Harnack inequalities, we prove some new sharp Gaussian type two-sided bounds for the heat kernels, which are new even for manifold M with nonnegative Ricci curvature. At the end, we discuss some open questions related to the sharp Li–Yau type estimates.