

# **Recent Progress on Representation Theory of Reductive Groups and Quantum Theory**

## **July 13-17 2025**

### **Organizing Committee**

Chengming Bai (Chern Institute of Mathematics, Nankai University)

Jing-Song Huang (The Chinese University of Hong Kong, Shenzhen)

Ning Li (Nankai University)

Kayue Daniel Wong (The Chinese University of Hong Kong, Shenzhen)

### **Supported by**

Chern Institute of Mathematics, Nankai University

Mathematics Research Center, The Chinese University of Hong Kong, Shenzhen

### **The schedule for the conference is as follows:**

July 13, 2025 (Sunday): Arrival and Registration

July 14-17, 2025: Research Presentations

July 17, 2025 (Thursday): Research Presentations and Departure

### **Contact Information**

Ning Li 18600663287 [lining@nankai.edu.cn](mailto:lining@nankai.edu.cn)

Guilai Liu 15940094170 [liugl@mail.nankai.edu.cn](mailto:liugl@mail.nankai.edu.cn)

Kayue Daniel Wong 17138691123 [danielwong@cuhk.edu.cn](mailto:danielwong@cuhk.edu.cn)

# **Information for participants**

## **Arrival and Registration Time:**

14:00-20:00, July 13, 2025

## **Place for Registration:**

Jiayuan Hotel, Nankai University

## **Meals**

Breakfast 7:15 a.m.-8:30 a.m.

Lunch (Buffet) 12:00 p.m.-13:00 p.m.

Dinner (Buffet) 18:00 p.m.-19:00 p.m.

There will be a conference banquet at 18:00pm, July 14th.

## **Conference Venue:**

Seminar room 216, Shiing-Shen Building

## **Wireless network at Chern Institute of Mathematics**

Wifi: NKU-WLAN, ID: cim-2025conf

# Invited Speakers

Dan Barbasch, Cornell University

Kei Yuen Chan, The University of Hong Kong

Taiwang Deng, Beijing Institute of Mathematical Sciences and Applications (BIMSA)

Chaoping Dong, Soochow University

Dongwen Liu, Zhejiang University

Jia-Jun Ma, Xiamen University

Kyo Nishiyama, Aoyama Gakuin University

Pavle Pandzic, University of Zagreb

Binyong Sun, Zhejiang University

Wei Xiao, Shenzhen University

Hirofumi Yamada, Rikkyo University

Penghui Yao, Nanjing University

Jun Yu, Peking University

Qing Zhang, Huazhong University of Science and Technology

Huangjun Zhu, Fudan University

# Schedule

**July 14, Monday**

Time	Speaker	Title	Host
8:10-8:40	Opening and Group Photo		
9:00-10:00	Binyong Sun	P-adic L-functions and modifying factors	Jing-Song Huang
10:00-10:30	Tea Break		
10:30-11:30	Jia-Jun Ma	Theta Correspondence over Finite Fields and its Relation to Quantum Codes	
12:00-14:00	Lunch and Break		
14:00-15:00	Chaoping Dong	Conjectures related to Dirac series	Daniel Wong
15:10-16:10	Qing Zhang	On product of Rankin-Selberg convolutions	
16:10-16:30	Tea Break		
16:30-17:30	Jun Yu	Positivity of Fourier transform of zonal spherical functions	
18:00	Dinner		

## July 15, Tuesday

Time	Speaker	Title	Host
9:30-10:30	Kei Yuen Chan	Schur-Weyl duality, Bernstein-Zelevinsky derivatives and Casselman-Jacquet functors	Shaoqiang Deng
10:30-11:00	Tea Break		
11:00-12:00	Kyo Nishiyama	Orbits on double flag varieties and quiver representations	
12:00	Lunch		
14:00-18:00	Free Discussion		

## July 16, Wednesday

Time	Speaker	Title	Host
9:30-10:30	Wei Xiao	Jantzen coefficients and intertwining differential operators on homogeneous vector bundles	Guilai Liu
10:30-11:00	Tea Break		
11:00-12:00	Pavle Pandzic	Clifford algebras and Littlewood-Richardson coefficients	
12:00-14:00	Lunch and Break		
14:00-15:00	Huangjun Zhu	The Clifford Group: Foundations and Applications in Quantum Information Science	Shizhuo Yu
15:10-16:10	Penghui Yao	Some Applications of Pauli analysis on Quantum Algorithms and Complexity	
16:10-16:30	Tea Break		
16:30-17:30	Hirofumi Yamada	Fock representations of the Virasoro algebra	
18:00	Dinner		

## July 17, Thursday

Time	Speaker	Title	Host
8:30-9:30	Dan Barbasch	Remarks on the comparison of the unitary dual of split vs quasi-split groups	Caihua Luo
9:40-10:40	Dongwen Liu	Blasius-Deligne conjecture for standard L-functions of symplectic type	
10:40-11:00	Tea Break		
11:00-12:00	Taiwang Deng	Orbital integrals of $GL_n$ over non-archimedean fields	
12:00	Lunch		
14:00-18:00	Departure		

# Title and Abstract

**Dan Barbasch (Cornell University)**

**Title:** Remarks on the comparison of the unitary dual of split vs quasi-split groups

**Abstract:** The spherical unitary dual of a split group is controlled very precisely by the relation between the Springer correspondence associating Weyl group representations to nilpotent orbits in the Lie algebra of the reductive group. A different relation between nilpotent orbits and Weyl group representations is established via the primitive ideal cells, study initiated by Jantzen, Joseph and Kazhdan-Lusztig. This difference affects in the shape of the unitary dual.

**Kei Yuen Chan (The University of Hong Kong)**

**Title:** Schur-Weyl duality, Bernstein-Zelevinsky derivatives and Casselman-Jacquet functors

**Abstract:** The classical Schur-Weyl duality relates the representation theory of symmetric groups and finite-dimensional  $GL_n(\mathbb{C})$ -representations. Recently, we constructed Schur-Weyl duality between the category of modules of graded Hecke algebras and the category of Harish-Chandra modules for  $GL_n(\mathbb{C})$ . Under such correspondence, the Bernstein-Zelevinsky derivative is transferred to the translation functor. On the other hand, methods are developed to compute information of simple quotients of Bernstein-Zelevinsky derivatives by Jacquet functors. This talk will explain how to compute those information of translation functors and its role in computing Casselman-Jacquet functors. It is based on some joint and ongoing work with Daniel Wong.

**Taiwang Deng (BIMSA)**

**Title:** Orbital integrals of  $GL_n$  over non-archimedean fields

**Abstract:** In 2014 Zhiwei Yun introduces a local L-factor whose special value at  $s=0$  computes the local orbital integral of type A. These L factors are controlled by the geometry of affine Springer fibers, more precisely the Hilbert scheme of points for the



related plane curve singularity. In this talk, we will explain a conjectural combinatorial formula of Cherednik for these L factors base on the affine paving for compactified Jacobians by Piontkowski. The affine paving is known to exist for compactified Jacobian in the generic case by work of Gorsky, Mazin and Oblomkov in 2024. Under the same setup, I will explain my ongoing work with Su Tao extending the previous work to show the existence of affine pavings for the Hilbert scheme of points and relating the affine cells to Young diagrams. As application, we apply our results to deduce a conjecture by Cherednik claiming that these local L-factors satisfy the Weil conjecture.

### **Chaoping Dong (SooChow University)**

**Title:** Conjectures related to Dirac series

**Abstract:** After introducing necessary background on Dirac cohomology, we will briefly introduce recent progresses on the classification of Dirac series. Then we will describe several conjectures pertaining to Dirac series. In particular, we will mention the recent work, joint with Chengyu Du and Haojun Xu, that any non-decreasable K-type is unitarily small.

### **Dongwen Liu (Zhejiang University)**

**Title:** Blasius-Deligne conjecture for standard L-functions of symplectic type

**Abstract:** Joint with Jiang, Sun and Tian, we prove the Blasius-Deligne conjecture for standard L-functions of symplectic type. This completes the project started in [JST19], by using different method to prove the archimedean period relations. In particular this includes the (almost complete) archimedean theory of Jacquet-Shalika integrals, which is used to realize the twisted Shalika models. The open orbit method plays a prominent role throughout this work.

### **Jia-Jun Ma (Xiamen University)**

**Title:** Theta Correspondence over Finite Fields and its Relation to Quantum Codes

**Abstract:** Let  $G_1$  and  $G_2$  be a reductive dual pair. The theta correspondence can be viewed as a functor from the category of representations of  $G_1$  to that of  $G_2$ . In

the first part of this talk, I will review the connection between the theory of theta correspondence and quantum codes, following the work of Montealegre-Mora and David Gross. In the second part, Zhe Li will present his results on the formalization of the definition and properties of the Weil representation of a double cover of the pseudo-symplectic group over a finite field of characteristic 2, following the work of Paull. We hope this expository talk will convince the audience that it is possible to study theta correspondence in characteristic 2 settings.

## **Kyo Nishiyama (Aoyama Gakuin University)**

**Title:** Orbits on double flag varieties and quiver representations

**Abstract:** Let  $G$  be a connected reductive algebraic group over  $\mathbb{C}$ . For a reductive subgroup  $L$  of  $G$ , we choose parabolic subgroups  $Q \subset L$  and  $P \subset G$  respectively. The product  $\mathfrak{X} = L/Q \times G/P$  with diagonal action of  $L$  is called a double flag variety. We are interested in those  $\mathfrak{X}$  with finitely many orbits.

In this talk, we consider Levi subgroups  $L$  of  $G$ , and explain how the representation theory of quivers can help to study the double flag variety and the orbits on it. As one of applications, we exhibit a complete classification of such double flag varieties with finitely many orbits when  $G$  is a general linear group.

The talk is based on the on-going joint work with Lucas Fresse.

## **Binyong Sun (IASM, Zhejiang University)**

**Title:** P-adic L-functions and modifying factors

**Abstract:** The Riemann zeta function  $\zeta(s)$  takes rational values at negative odd integers, and the Kummer congruence relations assert that these values satisfy congruences enabling them to be interpolated continuously in the p-adic topology. The p-adic L-function of Kubota-Leopoldt serves as the p-adic analog of the Riemann zeta function, incorporating the Kummer congruence relations. Coates conjectured that there exist p-adic L-functions attached to motives or algebraic automorphic representations, which interpolate the critical values of complex L-functions with explicit modifying factors at infinity and at  $p$ . Starting with basic notions on p-adic L-functions, we will discuss Coates' conjecture in the setting of automorphic representations. We verify it for symplectic-type representations. The talk is based on a joint work with Dongwen Liu.

## **Pavle Pandzic (University of Zagreb)**

**Title:** Clifford algebras and Littlewood-Richardson coefficients

**Abstract:** Let  $G/K$  be the compact symmetric space  $U(n)/U(p) \times U(n-p)$ . Let  $\mathfrak{p}$  be the orthogonal of the Lie algebra  $\mathfrak{k}$  of  $K$  in the Lie algebra  $\mathfrak{g}$  of  $G$  with respect to the Killing form. It is well known that the de Rham cohomology  $H(G/K)$  is isomorphic to  $(\wedge \mathfrak{p})^K$  as an algebra, and that it has a basis corresponding to Schur polynomials, with multiplication given by Littlewood-Richardson coefficients. We replace  $(\wedge \mathfrak{p})^K$  with its filtered version  $C(\mathfrak{p})^K$  which has a basis of orthogonal projections, with very simple multiplication. Relating the basis of projections with a filtered basis corresponding to Schur polynomials, one gets a new way to compute the Littlewood-Richardson coefficients.

## **Wei Xiao (Shenzhen University)**

**Title:** Jantzen coefficients and intertwining differential operators on homogeneous vector bundles

**Abstract:** We demonstrate that lower-degree intertwining differential operators on homogeneous vector bundles are determined by Jantzen coefficients. As a consequence, we provide a full classification of second-order intertwining differential operators on homogeneous line bundles. This talk is based on joint work with Yixin Bao.

## **Hirofumi Yamada (Rikkyo University)**

**Title:** Fock representations of the Virasoro algebra

**Abstract:** The Virasoro algebra is a central extension of the Lie algebra of vector fields on the circle  $S^1 = U(1)$ . Around 40 years ago, Minoru Wakimoto and I found an interesting relation between the Fock representations of the Virasoro algebra and the Schur functions. Our theorem is that rectangular Schur functions are the singular vectors of the Virasoro Fock representations. Soon after, Nolan Wallach realized the Virasoro algebra as vector fields on the unitary group  $U(n)$ , and gave another proof of our result. Recently Katsuhisa Mimachi (Osaka University) and I extended Wallach's

realization, and proved that rectangular Jack polynomials are Virasoro singular vectors.

### **Penghui Yao (Nanjing University)**

**Title:** Some Applications of Pauli analysis on Quantum Algorithms and Complexity

**Abstract:** Fourier analysis is playing a pivotal role in designing quantum algorithms. Recently, Fourier analysis on the space of operators and the space of super-operators, which is termed as Pauli analysis, has received increasing attention. It has found connections to various areas of quantum computing. In this talk, I will introduce some background on Pauli analysis and present some recently discovered applications in quantum learning theory and quantum complexity theory.

### **Jun Yu (Peking University)**

**Title:** Positivity of Fourier transform of zonal spherical functions

**Abstract:** Given a semisimple real linear group, a zonal spherical function is matrix coefficient associated to the unique spherical vector with value 1 at identity element in a unitary spherical principal series, which are important object in representation theory and harmonic analysis. Each zonal spherical function is a positive definite function. Hence, its Fourier transform along a maximal split torus is everywhere non-negative by a classical theorem of Salomon Bochner. In this talk we report a result in a recent joint work: the Fourier transform along a maximal split torus of any zonal spherical function takes positive value everywhere.

### **Qing Zhang (Huazhong University of Science and Technology)**

**Title:** On product of Rankin-Selberg convolutions

**Abstract:** Rankin-Selberg L-function of general linear groups is an important example of automorphic L-functions and it has many applications in number theory and Langlands program. In this talk I will report our construction of integrals which represent product of Rankin-Selberg L-functions of  $GL_\ell \times GL_m$  and  $GL_\ell \times GL_n$  if  $m + n < \ell$ . This talk is based on a joint work with Pan Yan.

## **Huangjun Zhu (Fudan University)**

**Title:** The Clifford Group: Foundations and Applications in Quantum Information Science

**Abstract:** The Clifford group is one of the most important groups in quantum information science. It consists of unitary operators that stabilize the Pauli group under conjugation. In this talk, we will first review the basic properties of the Clifford group and its deep connection with the symplectic group. Then, we will explore its wide-ranging applications in various research topics, including quantum computation, shadow estimation, entanglement theory, symmetric informationally complete measurements (SIC POVMs), mutually unbiased bases (MUB), and discrete Wigner functions. The talk will conclude with a discussion of open problems.