International Workshop on Bourgain-Demeter Decoupling Method

June 2-5，2019

Chern Institute of Mathematics

Nankai University

Tianjin, China

Scientific Committee:

Nicolas Burq, Universite Paris-Sud
Yiming Long, Nankai University
Changxing Miao, Institute of Applied Physics and Computational Mathematics
Christopher Sogge, Johns Hopkins University
Weiping Zhang, Nankai University

Organizing Committee:

Qionglei Chen, Institute of Applied Physics and Computational Mathematics
Wenchang Sun, Nankai University
Yuhua Sun, Nankai University
Yifei Wu, Tianjin University
Guixiang Xu, Beijing Normal University
Jiqiang Zheng, Institute of Applied Physics and Computational Mathematics

致谢：

会议得到下列单位资助和支持，在此深表谢意！

国家自然科学基金委员会

陈省身数学研究所

Information for participants of
International Workshop on Bourgain-Demeter Decoupling Method
Chern Institute of Mathematics，June 2-5, 2019

- Breakfast: 7:00-8:00 am,
  Lunch: 12:00-1:00 pm,
  Supper: 6:00-7:00 pm
- All lectures are arranged in the Seminar Room 216
- The main entrance of the Math. Library is on the 4th floor of the Shiing-Shen Building and can be used by conference participants. The Math. Library opens at 8:00-11:30am and 2:00-5:30pm during June 3-5.
- It is possible to connect personal computers to the Institute's network or to use the wireless network at CIM. (ID: guest, Password: cim20172017).
- You have to pay any phone call at the Jiayuan hotel. The network is free at the hotel.
- Bank：There is a commercial bank located near the east gate of Nankai University. ATM machines are accessible for overseas account via VISA, MASTER and PLUS.
- **Contact information**
  General office of the Chern Institute (Room 301 & 302)
  Tel: 2350 1029/ 2350 8228
  Jiayuan Hotel: Tel：0086-22-2350 8089
  Phone Number for calls to room NNN in Jiayuan Hotel:
  0086-22-23508089-ext. NNN or 0086-22-23509228-ext. NNN

PROGRAMME AT A GLANCE

Seminar Room 216, Chern Institute of Mathematics

	June 2	June 3	June 4	June 5
8:40—9:00	Opening Ceremony Group Photo			
	Chair: Changxing Miao	Chair: Christopher Sogge	Chair: Matthew Blair	Chair: Jean Saut
9:00—9:50	Christopher Sogge	Xiaochun Li	Alex Iosevich	Jonathan Hickman
Tea Break				
10:00—10:50	Guozhen Lu	Matthew Blair	Jason Metcalfe	Bobo Hua
Tea Break				
11:10—12:00	Yuan Zhou	Yifei Wu	Ruipeng Shen	Chuanwei Gao
Lunch				
	Chair: Guozhen Lu	Chair: Guixiang Xu	Chair: Xiaohua Yao	Chair: Jun Geng
14:00—14: 50	Xiumin Du	Shaoming Guo	Bochen Liu	Jianwei Yang
Tea Break				
15:00—15:50	Yuhua Sun	Changkeun Oh	Ting Chen	Kai Yang
	Tea Break			
16:00—16:50	Yakun Xi	Zehua Zhao	Danqing He	Cheng Zhang
Tea Break				
17:00—17:50	Dong Dong			
18:00	Dinner			

PROGRAMME

Seminar Room 216, Chern Institute of Mathematics

	June 2
08:40—09:00	Opening Ceremony, Group Photo
	Chair: **Changxing Miao**
09:00—09:50	**Christopher Sogge:** Quasimode estimates involving critically singular potentials
	Tea Break
10:00—10:50	**Guozhen Lu:** Fourier analysis on hyperbolic spaces and sharp geometric inequalities
	Tea Break
11:00—11:50	**Yuan Zhou:** A Sobolev $W^{2,2}$-regularity for parabolic normalized p-palace equaiton in plane
12:00	Lunch
	Chair: **Guozhen Lu**
14:00—14:50	**Xiumin Du:** Schr\"odinger maximal estimates and refined Strichartz type estimates
	Tea Break
15:00—15:50	**Yuhua Sun:** On positive solutions of semi-linear elliptic inequalities on Riemannian manifolds
	Tea Break
16:00—16:50	**Yakun Xi:** Distance sets on Riemannian surfaces and decoupling inequalities associated with Carleson-Sj\"olin phase
	Tea Break
17:00—17:50	**Dong Dong:** Discrete multilinear harmonic analysis and applications
18:00	Dinner

	June 3
	Chair: **Christopher Sogge**
09:00—09:50	**Xiaochun Li:** Roth's type theorems on (polynomial) progressions
	Tea Break
10:00—10:50	**Matthew Blair:** Logarithmic improvements in L^p bounds for eigenfunctions at the critical exponent in the presence of nonpositive curvature
	Tea Break
11:00—11:50	**Yifei Wu:** Some development on the estimates of the heat kernel
12:00	Lunch
	Chair: **Guixiang Xu**
14:00—14:50	**Shaoming Guo:** A proof of decoupling inequalities for the helix inspired by efficient congruencing
	Tea Break
15:00—15:50	**Changkeun Oh:** A bilinear proof of decouplings for some class of surfaces
	Tea Break
16:00—16:50	**Zehua Zhao:** Long time dynamics of threshold solutions for energy critical NLS
18:00	Dinner

	June 4
	Chair: **Matthew Blair**
09:00—09:50	**Alex Iosevich:** Configuration problems and applications to frame theory
	Tea Break
10:00—10:50	**Jason Metcalfe:** Local energy decay for wave equations in the presence of degenerate trapping
	Tea Break
11:00—11:50	**Ruipeng Shen:** Energy Distribution of Radial Wave Equations
12:00	Lunch
	Chair: **Xiaohua Yao**
14:00—14:50	**Bochen Liu:** Periodic structure of translational multi-tilings in the plane
	Tea Break
15:00—15:50	**Ting Chen:** Fractional integrals on Lebesgue spaces with mixed norms
	Tea Break
16:00—16:50	**Danqing He:** Bilinear multipliers with limited decay
18:00	Dinner

	June 5
	Chair: **Jean Saut**
09:00—09:50	**Jonathan Hickman:** The circular maximal function for Heisenberg radial functions
	Tea Break
10:00—10:50	**Bobo Hua:** Remarks on heat equations on manifolds and graphs
	Tea Break
11:00—11:50	**Chuanwei Gao:** Improved variable coefficient square functions and local smoothing of Fourier integral operators
12:00	Lunch
	Chair: **Jun Geng**
14:00—14:50	**Jianwei Yang:** Square function inequality for oscillatory integral operators satisfying homogeneous Carleson-Sj\"olin type conditions
	Tea Break
15:00—15:50	**Kai Yang:** Dynamics of focusing energy critical NLS
	Tea Break
16:00—16:50	**Cheng Zhang:** Restriction estimates of the eigenfunctions on Riemannian manifolds
18:00	Dinner

ABSTRACTS

Logarithmic improvements in L^p bounds for eigenfunctions at the critical exponent in the presence of nonpositive curvature

Matthew Blair

University of New Mexico

We consider the problem of determining upper bounds on the growth of L^p norms of eigenfunctions of the Laplacian on a compact Riemannian manifold in the high frequency limit. In particular, we seek to identify geometric or dynamical conditions on the manifold which yield improvements on the universal L^p bounds of C. Sogge. The emphasis here will be on bounds at the so-called "critical exponent" where one must rule out a spectrum of scenarios for phase space concentration in order to obtain an improvement. We then discuss a recent work with C. Sogge which shows that when the sectional curvatures are nonpositive, then there is a logarithmic type gain in the known L^p bounds at the critical exponent.

Fractional integrals on Lebesgue spaces with mixed norms

Ting Chen

Nankai University

Lebesgue spaces with mixed norms were first studied by Benedek and Panzone in 1961, where many fundamental properties were proved. In particular, they showed that such spaces possess similar properties as usual Lebesgue spaces. Recently, many works have been done for Lebesgue spaces with mixed norms. In this talk, we focus on the fractional integral opetrors on mixed Lebesgue spaces.

Discrete multilinear harmonic analysis and applications

Dong Dong

University of Maryland

We will first introduce a few classical multilinear operators in Euclidean space. Then we will turn to their discrete analogs. Here "discrete" refers to the setting of integer or finite field. Compared with the continuous case, the operators are in general more difficult to study in the integer setting, but are easier to handle in the finite field setting. These discrete operators have applications in PDE, ergodic theory, number theory, combinatorics, etc.

Schrödinger maximal estimates and refined Strichartz type estimates

Xiumin Du

University of Maryland

We consider Carleson's pointwise convergence problem of Schrödinger solutions. It is shown that the solution to the free Schrödinger equation converges to its initial data almost everywhere, provided that the initial data is in the Sobolev space $H^s(\mathbb{R}^n)$ with $s > n/2(n+1)$ (joint with Larry Guth and Xiaochun Li in the case $n = 2$, and joint with Ruixiang Zhang in the case $n > 2$). This is sharp up to the endpoint, due to a counterexample by Bourgain. This pointwise convergence problem can be approached by estimates of Schrödinger maximal functions. The key ingredients are refined Strichartz type inequalities derived from Bourgain–Demeter decoupling theorem and induction on scales.

Improved variable coefficient square functions and local smoothing of Fourier integral operators

Chuanwei Gao

Institude of Applied Physics and Computational Mathematics

Local smoothing conjecture which was formulated by Sogge has close relationship with other significant problems in Harmonic analysis and finds its extensive applications in PDEs. In this talk, we will present the recent improvement of local smoothing estimate for a certain class of FIOs in dimension 2 by means of bilinear approach. This is a joint work with C.Miao and J.Yang.

A proof of decoupling inequalities for the helix inspired by efficient congruencing

Shaoming Guo

University of Wisconsin-Madison

Bourgain, Demeter and Guth proved a sharp decoupling inequality for the helix. I will present a proof that is inspired by Wooley's method of efficient congruencing and Heath-Brown's simplified approach to Wooley's efficient congruencing. Joint work with Zane Li and Po-Lam Yung.

Bilinear multipliers with limited decay

Danqing He

Sun Yat-sen University

In this talk we plan to discuss some recent progress on maximal operators generated from bilinear multipliers with limited decay and some applications. The talk is based on

joint work with L. Grafakos, P. Honzík.

The circular maximal function for Heisenberg radial functions

Jonathan Hickman

University of St Andrews

We consider a variant of Bourgain's circular maximal function defined over the Heisenberg group H^1. Determining the L^p mapping properties of this operator is a challenging open problem and involves analysing a number of interesting singularities which are not present in the euclidean case. In recent joint work with David Beltran, Shaoming Guo and Andreas Seeger, the L^p mapping properties were determined under a radial assumption on the input function. Even under this assumption considerable difficulties arise, and the proof involves the analysis of a maximal function on the euclidean plane associated to a non-smooth curve distribution which fails both the rotational curvature and cinematic curvature conditions.

Remarks on heat equations on manifolds and graphs

Bobo Hua

Fudan University

In this talk, we discuss some properties of heat equations on manifolds and graphs. On one hand, Davies-Gaffney-Grigor'yan's lemma is a fundamental property of heat equations on manifolds, which fails on graphs in its original form. We present a variant of this lemma on graphs. On the other hand, we discuss heat equations on a covering of a manifold/graph, and give a geometric-analysis proof of Elworthy's theorem that the covering is stochastically complete if and only if so is the base, which was proved by the probability method. These are based on joint works with F. Bauer, F.Muench, S.T.-Yau, and R.K. Wojciechowski.

Configuration problems and applications to frame theory

Alex Iosevich

University of Rochester

We are going to describe a variety of connections between the existence of bases and frame of exponentials in Hilbert spaces and configuration problems in geometric measure theory and combinatorics, including the Erdos-Falconer distance problem and the Erdos integer distance principle. The connections between analysis, combinatorics and number theory will be emphasized throughout.

Roth's type theorems on (polynomial) progressions

Xiaochun Li

University of Illinois

The arithmetic progression problems were posed by Erdös-Turan, answered affirmatively by Semerédi. However, there are still many questions remained on precise quantitative description on how large a subset shall be in oredr to guarantee a progression in it. Involving with Fourier analysis, considerable work had been accomplished recently. We will give a survey on those progress, and report our recent progress on quantitative version of Roth's type theorem on (polynomial) progressions of short length.

Periodic structure of translational multi-tilings in the plane

Bochen Liu

The Chinese University of Hong Kong

A Borel set $P \subset \mathbb{R}^d$ is called a translational multi-tile if there exists a discrete set $\Lambda \subset \mathbb{R}^d$ such that

$$\sum_{\lambda \in \Lambda} \chi_P(x - \lambda)$$

is a constant almost everywhere. In this talk we will discuss about the existence of Λ and the periodic structure of Λ (if exists), given P is a convex polygon in the plane.

Fourier analysis on hyperbolic spaces and sharp geometric inequalities

Guozhen Lu

University of Connecticut

Sharp geometric inequalities play an important role in analysis and differential geometry. In this talk, we will review some recent works on sharp Hardy-Sobolev-Maz'ya inequalities on the upper half space which improve the classical Sobolev inequality. We will also discuss the borderline case of the Sobolev inequalities, namely, the Trudinger-Moser and Adams inequalities on hyperbolic spaces, Riemannian manifolds and Heisenberg groups. In particular, we will describe the Fourier analysis techniques on the hperbolic spaces and their applications to establish sharp geometric inequalities.

Local energy decay for wave equations in the presence of degenerate trapping

Jason Metcalfe

University of North Carolina

We will discuss local energy estimates for wave equations. Trapping is a known obstruction to such estimates. However, in the presence of unstable trapping, it was known that a logarithmic loss of regularity sufficed in order to recover the estimate. But when the

trapping was stable, all but a logarithmic amount of the decay is lost. Here we present the first explicit scenario where an algebraic loss of regularity is both necessary and sufficient. We moreover discuss the interaction of this metric with a boundary. In particular, the behavior bifurcates when the boundary passes through the trapping.

A bilinear proof of decouplings for some class of surfaces

Changkeun Oh

University of Wisconsin-Madison

This talk is about decouplings for some class of surfaces. This class of surfaces contains the surface of which decouplings are believed to be unable to be proved by Bourgain- Demeter's multilinear method. I will also present a short proof of a decoupling for paraboloid in $\mathbb{R}^3$, which is essentially due to Fan, Staffilani, Wang, and Wilson.

Energy Distribution of Radial Wave Equations

Ruipeng Shen

Tianjin University

The topic of this talk is the energy distribution property of solutions to a semi-linear, energy sub-critical, defocusing wave equation $\partial_t^2 u - \Delta u = -|u|^{p-1}u$ in the 3-dimensional space ($3 \leq p < 5$) whose initial data are radial. We split the energy into inward and outward energies, then apply energy flux formula to obtain the following property: Unless the solution scatters, its energy can be divided into two parts: "scattering energy" which concentrates around the light cone $|x| = |t|$ and moves to infinity at the light speed and "retarded energy" which also travels to infinity but is located at a distance of at least $|t|^\beta$ behind "scattering energy" when $|t|$ is large. Here β is an arbitrary constant smaller than $\beta_0(p) = \frac{2(p-2)}{p+1}$. A combination of this property with a more detailed version of the classic Morawetz estimate gives a scattering result under a weaker assumption on initial data (u_0, u_1) than previously known results.

Quasimode estimates involving critically singular potentials

Christopher Sogge

Johns Hopkins University

We prove eigenfunction and quasimode estimates on compact Riemannian manifolds for Schrödinger operators, $H_V = -\Delta_g + V$ involving critically singular potentials V which we assume to be in $L^{n/2}$ and/or the Kato class $\mathcal{K}$. Our proof is based on modifying the oscillatory integral/resolvent approach that was used to study the case where $V \equiv 0$ using recently developed techniques by many authors to study variable coefficient analogs of the uniform Sobolev estimates of Kenig, Ruiz and the speaker. Using the quasimode estimates we are able to obtain Strichartz estimates for wave equations. We are also

able to prove corresponding results for Schrödinger operators in $\mathbb{R}^n$ and can obtain a natural generalization of the Stein-Tomas restriction theorem involving potentials with small $L^{n/2}(\mathbb{R}^n)$ norms.

This is joint work with M. Blair and Y. Sire.

On positive solutions of semi-linear elliptic inequalities on Riemannian manifolds

Yuhua Sun

Nankai University

We determine the critical exponent for certain semi-linear elliptic problem on a Riemannian manifold assuming the volume regularity and Green function estimates. This is based on the joint work with Prof. A. Grigor'yan.

Some development on the estimates of the heat kernel

Yifei Wu

Tianjin University

The maximal regularity for the heat kernel plays an essential roles in the study on the maximal regularity of solutions to the nonlinear parabolic equations. In this talk, we give some discussion on this type of estimates, and present some details of the proof.

Distance sets on Riemannian surfaces and decoupling inequalities associated with Carleson-Sjölin phase

Yakun Xi

University of Rochester

In this talk, we discuss the generalization of the Falconer distance problem to the Riemannian setting. In particular, we extend the recent result of Guth-Iosevich-Ou-Wang for the distance set in the plane to general Riemannian surfaces. The key new ingredient is a family of refined decoupling inequalities associated to phase functions that satisfy Carleson-Sjölin condition. This is a joint work with Iosevich and Liu.

Square function inequality for oscillatory integral operators satisfying homogeneous Carleson-Sjölin type conditions

Jianwei Yang

Université Paris 13

We establish an improved square function inequality for a class of Hörmander-type oscillatory integrals satisfying homogeneous Carleson-Sjölin conditions. As a consequence,

we further improve the $L^p \to L^p$local smoothing estimate with $p\Phi(2,6)$ for Fourier integral operators satisfying cinematic curvature conditions in dimension two. The main ingredients in the argument consists of multilinear oscillatory integral estimate of Bennett-Carbery-Tao (Acta Math., 196(2):261–302, 2006) and decoupling inequality of Beltran-Hickman-Sogge (ArXiv preprint 1801.06910v1).

This work is joint with Chuanwei Gao and Changxing Miao.

Dynamics of focusing energy critical NLS

Kai Yang

Southeast University

In this talk, we will discuss recent development of the dynamics of the focusing energy critical NLS. We will present new approach to the non-radial setting of this problem for sub-threshold solutions in high dimensions. We also consider the initial value problem for the focusing energy critical NLS with an inverse square potential. We give a full characterization of all solutions when the energy of the solutions equals the energy of the ground state. This is joint work with Prof. Dong Li and Prof. Xiaoyi Zhang.

Restriction estimates of the eigenfunctions on Riemannian manifolds

Cheng Zhang

Johns Hopkins University

We will introduce some recent results on the endpoint restriction estimates of the eigenfunctions on Riemannian manifolds, including the latest results on the compact hyperbolic manifolds and the flat torus. These improve the general results by Burq-Gerard-Tzvetkov(2007), and are related to a conjecture of Bourgain-Rudnick(2011).

Long time dynamics of threshold solutions for energy critical NLS

Zehua Zhao

Johns Hopkins University

In this talk, we discuss the dynamics of subcritical threshold solutions for focusing energy critical NLS on $\mathbb{R}^d$ $(d \geq 5)$ with nonradial data. This problem with radial assumption was studied by T. Duyckaerts and F. Merle for $d = 3, 4, 5$ and later by D. Li and X. Zhang for $d \geq 6$. We generalize the conclusion for the subcritical threshold solutions by removing the radial assumption for $d \geq 5$. A key step is to show exponential convergence to the ground state $W(x)$ up to symmetries if the scattering phenomenon does not occur. Remarkably, an interaction Morawetz-type estimate is applied. The talk is based on a joint work with Qingtang Su (University of Michigan). We will introduce the background and existing results related to this problem. Then we will talk about the main idea of our result.

A Sobolev $W^{2,2}$-regularity for parabolic normalized p-palace equaiton in plane

Yuan Zhou

Beihang University

For $n = 2$ and $p \in (1, \infty)$, if $u = u(x, t)$ is a viscosity solution to the parabolic normalized p-Laplace equation

$$\frac{\partial u}{\partial t} - |Du|^{2-p}\mathrm{div}(|Du|^{p-2}Du) = 0,$$

we prove that u enjoys a quantative $W^{2,2}_{loc}$-regularity in spatial variables and $W^{1,2}_{loc}$-regularity in time variable. This answers an open question raised by Høeg and Lindqvist when $n = 2$. Moreover, this also gives a new proof for the $W^{2,2}_{loc}$-regularity of p-harmonic function when $n = 2$. This is a joint work with Peng Fa.

List of Participants

Matthew Blair	University of New Mexico
Qionglei Chen	Institute of Applied Physics and Computational Mathematics
Ting Chen	Nankai University
Yu Chen	Graduate School of China Academy of Engineering Physics
Xing Cheng	Hohai University
Hongcun Deng	Yantai University
Mingming Deng	Graduate School of China Academy of Engineering Physics
Dong Dong	University of Illinois
Xiumin Du	University of Maryland
Chuanwei Gao	Institute of Applied Physics and Computational Mathematics
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Jun Geng	Lanzhou University
Jingwei Guo	University of Science and Technology of China
Shaoming Guo	University of Wisconsin-Madison
Xiaonan Hao	Institute of Applied Physics and Computational Mathematics
Danqing He	Sun Yat-sen University
Daoyin He	Fudan University
Jonathan Hickman	University of St Andrews
Bobo Hua	Fudan University
Shanlin Huang	Central China Normal University
Alex Iosevich	University of Rochester
Renjin Jiang	Tianjin University
Dan Li	Beijing Normal University
Jingyue Li	Institute of Applied Physics and Computational Mathematics
Junfeng Li	Beijing Normal University
Wenjuan Li	Northwestern Polytechnical University
Xiang Li	Beijing Normal University
Xiaochun Li	University of Illinois
Yatao Li	Graduate School of China Academy of Engineering Physics
Zhuoran Li	Institute of Applied Physics and Computational Mathematics
Bochen Liu	The Chinese University of Hong Kong
Guozhen Lu	University of Connecticut
Fanfei Meng	Graduate School of China Academy of Engineering Physics
Jason Metcalfe	University of North Carolina
Changxing Miao	Institute of Applied Physics and Computational Mathematics
Changkeun Oh	University of Wisconsin-Madison
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Christopher Sogge	Johns Hopkins University
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Jianwei Yang	Beijing Institute Of Technology
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Xiaohua Yao	Central China Normal University
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Cheng Zhang	Johns Hopkins University
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