

# **International Workshop on Commutative Algebra and Related Topics**

**September 17-20, 2024**



**CHERN INSTITUTE OF MATHEMATICS**

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南开大学陈省身数学研究所

**International Workshop on  
Commutative Algebra  
and Related Topics**

**Program**

**September 17-20, 2024**

**Chern Institute of Mathematics**

**Tianjin, China**

## **Organizers**

Zhongming Tang (Suzhou University), Chair

Ngo Viet Trung (Vietnam Academy of Sciences and Technology)

Siamak Yassemi (Purdue University)

Cheng Gong (Suzhou University)

## **Supported by**

Chern Institute of Mathematics, Nankai University

## Program at A Glance

	Tuesday	Wednesday	Thursday	Friday
9:00-9:45	Bernd Ulrich	Le Tuan Hoa	Gunnar Floeystad	Dale Cutkosky
9:55-10:40	Tim Roemer	Giulio Caviglia	Volkmar Welker	Ha Huy Tai
10:40-11:05	Tea Break	Tea Break	Tea Break	Tea Break
11:05-11:50	Yi-Huang Shen	Cheng Meng	Cheng Gong	A. V. Jayanthan
12:00-14:00	Lunch	Lunch	Lunch	Lunch
14:00-14:45	Aldo Conca	Marc Chardin	Sijong Kwak	Claudia Polini
14:55-15:40	Manolis Tsakiris	Alexandru Constantinescu	Hema Srinivasan	I-Chiau Huang
15:40-16:05	Tea Break	Tea Break	Tea Break	Tea Break
16:05-16:50	Nguyen Dang Hop	Jin Guo	Tran Nam Trung	Vivek Mukundan
17:00-17:45	Dancheng Lu	Do Trong Hoang	Guangjun Zhu	Jugal Verma
18:00-20:00			Banquet	

All the lectures take place in Lecture Hall 216, Shiing-Shen Building

**September 16, Monday**

10:00-20:00 **Registration** at Jiayuan Hotel, Nankai University

**September 17, Tuesday**

Location: Lecture Hall 216, Shiing-Shen Building

8:45-9:00 **Opening Ceremony** (Host: **Zhongming Tang** (Suzhou University))

**Chair of the morning session: Ngo Viet Trung** (Vietnam Academy of Sciences and Technology)

9:00-9:45 **Bernd Ulrich** (Purdue University, USA)

*Ideals in the linkage class of a complete intersection: recent results and conjectures*

9:55-10:40 **Tim Roemer** (Osnabrück University, Germany)

*Symmetries and local-global principles in discrete geometry*

10:40-11:05 **Tea Break**

11:05-11:50 **Yi-Huang Shen** (University of Science and Technology of China, China)

*Powers of generalized binomial edge ideals of path graphs*

12:00-14:00 **Lunch time**

**Chair of the afternoon session: Volkmar Welker** (University of Marburg, Germany)

14:00-14:45 **Aldo Conca** (University of Genova, Italy)

*Bounds on Castelnuovo-Mumford regularity*

14:55-15:40 **Manolis Tsakiris** (Chinese Academy of Sciences, China)

*A family of subspace arrangements with linear resolution*

15:40-16:05 **Tea Break**

16:05-16:50 **Nguyen Dang Hop** (Vietnam Academy of Sciences and Technology, Vietnam)

*Asymptotic regularity and projective dimension of symmetric chains of monomial ideals*

17:00-17:45 **Dancheng Lu** (Suzhou University, China)

*The resolutions of generalized co-letterplace ideals and their powers*

**September 18, Wednesday**

Location: Lecture Hall 216, Shiing-Shen Building

**Chair of the morning session: Jugal Verma** (Indian Institute of Technology, Gandhinagar, India)

9:00-9:45 **Le Tuan Hoa** (Vietnam Academy of Sciences and Technology, Vietnam)

*Initial behavior of maximal generating degrees and their stability indexes*

9:55-10:40 **Giulio Caviglia** (Purdue University, USA)

*Bounds on the arithmetic degree*

10:40-11:05 **Tea Break**

11:05-11:50 **Cheng Meng** (Tsinghua University, China)

*Lech's conjecture on multiplicities in flat local extensions*

12:00-14:00 **Lunch time**

**Chair of the afternoon session: Dale Cutkosky** (University of Missouri, USA)

14:00-14:45 **Marc Chardin** (Sorbonne University, France)

*Cohomology and resolution of points in a product of projective spaces*

14:55-15:40 **Alexandru Constantinescu** (Freie Universität Berlin, Germany)

*Castelnuovo-Mumford regularity versus virtual cohomological dimension*

15:40-16:05 **Tea Break**

16:05-16:50 **Jin Guo** (Hainan University, China)

*On the primality of collections of cells*

17:00-17:45 **Do Trong Hoang** (Hanoi University of Science and Technology, Vietnam)

*Depth and regularity of tableau ideals*

**September 19, Thursday**

Location: Lecture Hall 216, Shiing-Shen Building

**Chair of the morning session: Aldo Conca** (University of Genova, Italy)

9:00-9:45 **Gunnar Floeystad** (University of Bergen, Norway)

*Polarizations of Artin monomial ideals define triangulated balls*

9:55-10:40 **Volkmar Welker** (University of Marburg, Germany)

*On subadditivity of shifts*

10:40-11:05 **Tea Break**

11:05-11:50 **Cheng Gong** (Suzhou University, China)

*The geometry of syzygies and their applications*

12:00-14:00 **Lunch time**

**Chair of the afternoon session: Claudia Polini** (University of Notre Dame, USA)

14:00-14:45 **Sijong Kwak** (Korea Advanced Institute of Science and Technology (KAIST), Korea)

*Introduction to higher secant varieties of minimal degree and their characterizations*

14:55-15:40 **Hema Srinivasan** (University of Missouri, USA)

*Numerical semigroups, critical relations and principal matrices*

15:40-16:05 **Tea Break**

16:05-16:50 **Tran Nam Trung** (Vietnam Academy of Sciences and Technology, Vietnam)

*Regularity of symbolic powers of square-free monomial ideals*

17:00-17:45 **Guangjun Zhu** (Suzhou University, China)

*Edge ideals of some edge-weighted graphs*

**September 20, Friday**

Location: Lecture Hall 216, Shiing-Shen Building

**Chair of the morning session: Bernd Ulrich** (Purdue University, USA)

9:00-9:45 **Dale Cutkosky** (University of Missouri, USA)

*Rees algebras and analytic spread of graded filtrations*

9:55-10:40 **Ha Huy Tai** (Tulane University, USA)

*Resurgence of pairs of graded families of ideals*

10:40-11:05 **Tea Break**

11:05-11:50 **A. V. Jayanthan** (Indian Institute of Technology, Madras, India)

*On the unmixedness and Cohen-Macaulayness of the parity binomial edge ideals*

12:00-14:00 **Lunch time**

**Chair of the afternoon session: Zhongming Tang** (Suzhou University)

14:00-14:45 **Claudia Polini** (University of Notre Dame, USA)

*Infinite free resolutions: a finite story for Golod rings*

14:55-15:40 **I-Chiau Huang** (Academia Sinica)

*Gluings and fibered sums of affine semigroups*

15:40-16:05 **Tea Break**

16:05-16:50 **Vivek Mukundan** (Indian Institute of Technology, Delhi, India)

*Berger's Conjecture and reduced type of one dimensional rings*

17:00-17:45 **Jugal Verma** (Indian Institute of Technology, Gandhinagar, India)

*Hilbert-Kunz multiplicity of powers of an ideal*

17:45-17:55 **Closing Ceremony**



## **Titles and Abstracts**

**Giulio Caviglia**

***Bounds on the arithmetic degree***

In local commutative algebra, certain arguments, such as the existence of a linear system of parameters or superficial sequences, typically require the residue field to be infinite. In the first half of the talk I will show how to use upper bounds on the arithmetic degree of graded rings, for instance the ones proved by Yihui Liang, to derive (lower) bounds for the cardinality of the residue field over which these arguments still hold. In the second half of the talk I will show that a finitely generated graded module  $M$  over  $S = k[x_1, \dots, x_n]$ ,  $k$  a field, is sequentially Cohen-Macaulay if and only if its arithmetic degree  $\text{adeg}(M)$  agrees with  $\text{adeg}(F/\text{gin}_{\text{revlex}}(U))$ , where  $F$  is a graded free  $S$ -module and  $M \cong F/U$ . This answers positively a conjecture of Lu and Yu from 2016.

**Marc Chardin**

***Cohomology and resolution of points in a product of projective spaces***

We will report on joint work with Navid Nemati and Tran Quang Hoa exploring the shape and duality properties of cohomology and Betti numbers for points in a product of two projective spaces. A particular focus will be developed concerning points that are defined by as many equation as the dimension of the space.

**Aldo Conca**

***Bounds on Castelnuovo-Mumford regularity***

I will present two results that explore bounds on the Castelnuovo-Mumford regularity. The first result is a regularity bound for ideals with polynomial parametrization in terms of the degree of the polynomials involved as well as their number and the number of variables. This is topic I have investigated in collaboration with Francesca Cioffi (Napoli). The second result deals with ideals associated with general subspace arrangements. This research, conducted jointly with Manolis Tsakiris (Chinese Academy of Sciences), offers new perspectives on the regularity of these ideals. Indeed we establish upper bounds, improving earlier results of Derksen and Sidman as well as lower bounds.

**Alexandru Constantinescu**

***Castelnuovo-Mumford regularity versus virtual cohomological dimension***

This talk is based on a surprising connection between a question in commutative algebra regarding Castelnuovo-Mumford regularity and a question posed by Gromov about hyperbolic Coxeter groups. We show that the virtual cohomological dimension of a Coxeter group is essentially the same as the Castelnuovo-Mumford regularity of the Stanley-Reisner ring of its nerve. Using this relation, we modify a construction of Osajda in group theory to find, for every positive integer  $r$ , a monomial ideal generated in degree two, with linear syzygies, and regularity of the quotient equal to  $r$ . Previously known examples had regularity less than 5. For Gorenstein ideals, we proved that the regularity of their quotients cannot exceed four, thus showing that for  $d \geq 4$ , every triangulation of a  $d$ -manifold has a hollow square or simplex. We also showed that for most monomial ideals generated in degree two and with linear syzygies, the regularity is  $O(\log(\log(n)))$ , where  $n$  is the number of variables. All results were obtained in collaboration with Thomas Kahle and Matteo Varbaro.

## **Dale Cutkosky**

### ***Rees algebras and analytic spread of graded filtrations***

We present recent joint work by the author and recent joint work with Parangama Sarkar showing that much of the classical theory for  $I$ -adic filtrations does extend to general graded filtrations or at least to divisorial filtrations. We discuss examples for which classical results do not extend and interesting ways in which the theory is different.

## **Gunnar Floeystad**

### ***Polarizations of Artin monomial ideals define triangulated balls***

Given an Artin monomial ideal  $I$  in a polynomial ring  $k[x_1, x_2, \dots, x_n]$ . So for each variable  $x_i$  some power  $x_i^{a_i} \in I$ . Let  $f: A \rightarrow \{x_1, \dots, x_n\}$  be a map of finite sets with fibers  $A_i = f^{-1}(x_i)$  of cardinality  $a_i$ , and let  $\Delta(A_i)$  be the simplex with vertices  $A_i$ . A polarization ideal  $J$  of  $I$  is a squarefree monomial ideal in the polynomial ring  $k[A]$  with variables the elements of  $A$ , with an induced map of quotient rings  $k[A]/J \rightarrow k[x_1, \dots, x_n]/I$  such that the latter quotient ring is obtained from  $k[A]/J$  by dividing out by a regular sequence of variable differences. Proving a conjecture of A.Almousa, G.Floeystad, and H.Lohne, we show:

- Any such polarization  $J$  defines, via the Stanley-Reisner correspondence, a triangulated ball  $B$ .

- $B$  is a full dimensional subcomplex of the boundary of the join polytope  $\Delta(A_1) * \Delta(A_2) * \cdots * \Delta(A_n)$ .
- The Alexander dual ideal  $J^\vee$  has generators corresponding to vertices of the product space  $\Delta(A_1) \times \Delta(A_2) \times \cdots \times \Delta(A_n)$ .

These vertices define a sub-cellcomplex of this product space, giving a linear cellular resolution of the Alexander dual ideal  $J^\vee$ .

## **Cheng Gong**

### ***The geometry of syzygies and their applications***

In this talk, we will introduce the classifications of syzygies of points and curves, and further we will introduce their applications to algebraic geometry and dynamical systems. In particular, we will discuss multiplier ideals and the stability of systems.

## **Jin Guo**

### ***On the primality of collections of cells***

For a polyomino or a collection of cells, to determine the primality of its polyomino ideal is an important topic in combinatorial commutative algebra. In this talk, we introduce some recent works about this topic. On one hand, there are some structures with which the polyominoes are not prime; On the other hand, with some special structures, many polyominoes are proved to be prime. Furthermore, we introduce a new method called zero-sum condition. Based on this method, we construct a class of prime polyominoes, each of which contains a good rectangle piece. We also give a class of prime collections of cells called extended bipartite collections of cells. These two classes almost include all the known prime polyominoes and prime collections of cells.

## **Le Tuan Hoa**

### ***Initial behavior of maximal generating degrees and their stability indexes***

Given a homogeneous ideal  $I$  of a polynomial ring  $R$ , it is well-known that the maximal generating degree  $d(I^n)$  of  $I^n$  is a linear function  $p(I)n + q(I)$  for all  $n \gg 0$ . The difference  $E_{\deg}(I; n) := d(I^n) - p(I)n$  is called the degree excess function of  $I$ . Similarly, the difference  $\overline{E}_{\deg}(I; n) := d(\overline{I}^n) - p(I)n$  is called the normal degree excess function of  $I$ , where  $\overline{I}^n$  denotes the integral closure of  $I^n$ . In this talk, I present some results on the initial

behavior of these functions. In the case of monomial ideals, I also give bounds on the value, from which these functions become linear.

## **Do Trong Hoang**

### ***Depth and regularity of tableau ideals***

A Young diagram  $Y$  is a collection of boxes arranged in left-justified shape of  $n$  rows of boxes of length  $\lambda_1, \dots, \lambda_n$  with  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ . A *filling* of a Young diagram is defined as any assignment of positive integers  $\omega(i, j)$  to each box  $(i, j)$  in the diagram. The *tableau ideal* associated with Young diagram  $Y$  with a given filling is an ideal in a standard graded polynomial ring  $S = k[x_1, \dots, x_n, y_1, \dots, y_m]$  over a field  $k$ , where  $m = \lambda_1$ , and is defined as follows:

$$I(Y) = ((x_i y_j)^{\omega(i, j)} \mid 1 \leq i \leq n, 1 \leq j \leq \lambda_i).$$

In this talk, we will present a simple recursive formula for computing the depth and regularity of  $I(Y)$ . These stem from a joint work with Thanh Vu.

## **Nguyen Dang Hop**

### ***Asymptotic regularity and projective dimension of symmetric chains of monomial ideals***

We consider the asymptotic behavior of chains of monomial ideals that are stable under the action of the monoid  $\text{Inc}$  of increasing functions  $Z_{>0} \rightarrow Z_{>0}$ . It is conjectured that for such chains, the regularity and projective dimension are eventually linear functions. We confirm the conjecture and provide complete description of the regularity and projective dimension (equivalently, the depth) in the case of chains of edge ideals. Remarkably, if the ideals in the chain are non-zero, then the regularity function is eventually constant with only two possible limiting values, and the same thing happens for the depth. Our results and their proofs also reveal many interesting combinatorial and topological properties of  $\text{Inc}$ -invariant chains of graphs and their independence complexes. Joint work with Tran Quang Hoa, Do Trong Hoang, Dinh Van Le, and Thai Thanh Nguyen.

## **I-Chiau Huang**

### ***Gluings and fibered sums of affine semigroups***

I will talk about the existence of gluings and its relation with fibered sums. This is a joint work with C-Y. Jean Chan and Jung-Chen Liu. Our interests on the subject began by extending the relative viewpoint from numerical semigroups to affine semigroups. The current work is motivated by a question posed by Gimenez and Srinivasan [3] regarding when and how affine semigroups can be glued. For numerical semigroups, gluing has been extensively studied. It appeared in Watanabe [16, Lemma 1] in a special case following the study of complete intersection numerical semigroup rings by Herzog [7]. Delorme [1] treated gluing systematically for numerical semigroups and obtained a thorough characterization for complete intersections. Gluing was generalized to affine semigroups by Rosales [13]. It was further studied in Gimenez and Srinivasan [3, 4].

I will talk about a particular diagram, smallest in the sense that all such diagrams emerge from it. So what we construct is a universal object among these diagrams, or in other words, a fibered sum in the category of affine semigroup rings. Our construction will be compared to the tensor product and to gluings of affine semigroup rings. While gluings of affine semigroup rings do not always exist, fibered sum can always be achieved. We investigate when the fibered sum of affine semigroup algebras gives rise to a gluing. A criterion for the existence of gluing is recovered.

### **A. V. Jayanthan**

#### ***On the unmixedness and Cohen-Macaulayness of the parity binomial edge ideals***

In this talk, we explore the Cohen-Macaulayness of the parity binomial edge ideals of chordal graphs in terms of certain structural properties of the graphs. While the binomial edge ideals and parity binomial edge ideals look similar, we show that their unmixed properties are very different. This is an ongoing joint work with Deblina Dey and Sarang Sane.

### **Vivek Mukundan**

#### ***Berger's Conjecture and reduced type of one dimensional rings***

Let  $R$  be a domain that is a complete local  $k$  algebra in dimension one. In an effort to address the Berger's conjecture, a crucial invariant reduced type  $s(R)$  was introduced by Huneke. In this talk, we study this invariant and its max/min values separately and relate it to the valuation semigroup of  $R$ . We justify the need to study  $s(R)$  in the context of numerical semigroup rings and consequently investigate the occurrence of the extreme values of  $s(R)$  for the Gorenstein, almost Gorenstein, and far-flung Gorenstein complete

numerical semigroup rings. Finally, we study the finiteness of the category of maximal Cohen Macaulay modules and the category of reflexive modules for rings which are of maximal/minimal reduced type and provide many classifications.

### **Sijong Kwak**

#### ***Introduction to higher secant varieties of minimal degree and their characterizations***

Projective varieties of minimal degree are one of the basic objects in projective algebraic geometry and syzygies. They usually appears as boundary cases in some interesting results, e.g., the  $K_{p,1}$  Theorem, the syzygetic Castelnuovo Lemma, etc. These varieties are completely classified due to del Pezzo (1886) and Bertini (1907) with an centennial account due to D. Eisenbud and J. Harris (1987). Projective varieties of minimal degree have also geometric, syzygetic and determinantal characterizations.

In this talk, we introduce the higher secant varieties of minimal degree due to C. Ciliberto and F. Russo (2016) to nonexperts with modest background. We would like to describe those varieties in terms of syzygy and geometry with determinantal presentations. This is a joint work with Dr. Junho Choe in KAIST.

### **Dancheng Lu**

#### ***The resolutions of generalized co-letterplace ideals and their powers***

In this talk, we will present a natural and explicit multigraded minimal free resolution for each generalized co-letterplace ideal. Our resolution differs significantly from the ones presented in the works of Ene et al. [2] and D'Alì et al. [1]. Additionally, we show that each power of a large class of generalized co-letterplace ideals can be represented as the quotient of another generalized co-letterplace ideal by a regular sequence of variable differences. Finally, we provide a new class of simplicial spheres.

[1] D'Alì, A., Fløystad, G., & Nematbakhsh, A. (2019). Resolutions of co-letterplace ideals and generalizations of bier spheres. Transactions of the American Mathematical Society, 371(12), 8733-8753.

[2] Ene, V., Herzog, J. & Mohammadi, F. (2011). Monomial ideals and toric rings of Hibi type arising from a finite poset. European Journal of Combinatorics, 32(3), 404-421.

### **Cheng Meng**

#### ***Lech's conjecture on multiplicities in flat local extensions***

In this talk, we will introduce Lech's conjecture, which states that in a flat local extension, the Hilbert-Samuel multiplicity of the extension ring is no less than the multiplicity of the base ring. We will talk about the notion of strongly Lech-independent ideals and use this notion to derive inequalities on multiplicities of ideals. In particular, we prove that if in the flat local extension, the rings have the same dimension, the extension ring is standard graded over a field, and the extension of the maximal ideal of the base ring is homogeneous, then Lech's conjecture holds.

**Claudia Polini**

***Infinite free resolutions: a finite story for Golod rings***

We prove a surprising finiteness result for Golod rings: All the syzygies of the residue field of a Golod ring are direct sums of a finite set of indecomposable modules. This finitistic behavior is diametrically opposite to the behavior of Artinian Gorenstein rings, where each syzygy of the residue field is indecomposable. In addition, we describe in detail the case of any local ring of embedding dimension two that is not a complete intersection. In this case, all syzygy modules are direct sums of only three possible modules, the residue field, the maximal ideal, and the dual of the maximal ideal.

**Tim Roemer**

***Symmetries and local-global principles in discrete geometry***

We study cones and other objects of interest in discrete geometry, but here in infinite dimensional spaces which are invariant under actions of symmetric groups. Our approach is to associate to such objects certain symmetric chains of cones, etc. in finite dimensional spaces. Then via local-global principles one can understand global objects in infinite dimensional spaces via local ones in the associated chains. In this talk we discuss recent results and open questions on these topics.

**Yi-Huang Shen**

***Powers of generalized binomial edge ideals of path graphs***

We study the powers of the generalized binomial edge ideal  $J_{K_m, P_n}$  of a path graph  $P_n$ . We explicitly compute their regularities and determine the limit of their depths. We also show that these ordinary powers coincide with their symbolic powers. Additionally, we study the Rees algebra and the special fiber ring of  $J_{K_m, P_n}$  utilizing Sagbi basis theory. In

particular, we obtain precise formulas for the regularity of these blowup algebras. This is a joint work with Guangjun Zhu.

**Hema Srinivasan**

***Numerical Semigroups, Critical Relations and Principal Matrices***

A Numerical semigroup  $S$  is a submonoids of Natural Numbers minimally generated by a set of  $n$  relatively prime positive integers, say,  $a_1, a_2, \dots, a_n$ . The number  $n$  is called the embedding dimension of the semigroup  $S$ . The ring  $k[t^i | i \in S]$  is called the semigroup ring associated to  $S$  and is isomorphic to  $k[x_1, \dots, x_n]/I_S$ , where  $I_S$  is a binomial prime ideal of height  $n-1$ . There are special binomials in  $I_S$  called critical binomials which are represented in an  $n \times n$  matrix called the principal matrix of  $S$ . We will introduce the problem of determining  $S$  from its principal matrix and study the structure of  $S$  from a principal matrix discuss some results.

**Ha Huy Tai**

***Resurgence of pairs of graded families of ideals***

Resurgence and asymptotic resurgence numbers of an ideal were introduced in the study of the Ideal Containment Problem, as measures for the non-containment between symbolic and ordinary powers of the ideal. We will introduce generalized notions associated to a pair of graded families of ideals and discuss the questions of when these invariant are finite, computable, and how to understand them combinatorially.

**Manolis Tsakiris**

***A family of subspace arrangements with linear resolution***

The now classical bound of Derksen & Sidman (2002) asserts that the ideal that defines the reduced union of  $n$  linear spaces in projective space has Castelnuovo-Mumford regularity at most  $n$ . For unions of generic points or generic lines the actual regularity, read from the available in these cases Hilbert function, is known to be much smaller than  $n$ . On the other hand, for intersecting spaces, and in particular for spaces of small codimension, the picture has been unclear (besides the trivial case of hyperplanes). This talk will discuss a family of arrangements of  $n$  linear spaces of codimension 2, parameterized by the dimension of the ambient projective space, whose ideal has regularity  $n-1$  and a linear resolution; the Hilbert function will also be described. Connections with the Hartshorne-Hirschowitz theorem (1981) will be drawn. This is joint work with Aldo Conca.



**Tran Nam Trung**

***Regularity of symbolic powers of square-free monomial ideals***

Let  $K$  be a field and  $R = K[x_1, \dots, x_r]$  the polynomial ring of  $r$  variables  $x_1, \dots, x_r$  with  $r \geq 1$ . Let  $I$  be a homogeneous ideal of  $R$ . Then the  $n$ -th symbolic power of  $I$  is defined by

$$I^{(n)} = \bigcap_{P \in \text{Min}(I)} I^n R_P \cap R,$$

where  $\text{Min}(I)$  is as usual the set of minimal associated prime ideals of  $I$ . We study the regularity of symbolic powers of square-free monomial ideals. We prove that if  $I = I_\Delta$  is the Stanley-Reisner ideal of a simplicial complex  $\Delta$ , then  $\text{reg}(I^{(n)}) \leq \delta(n-1) + b$  for all  $n \geq 1$ , where  $\delta = \lim_{n \rightarrow \infty} \text{reg}(I^{(n)})/n$ , and  $b = \max \left\{ \text{reg}(\sqrt{I^{(n)}} : f \mid f \text{ is a monomial with } f \notin I^{(n)}) \right\}$ . This bound is sharp for any  $n$ . When  $I = I(G)$  is the edge ideal of a simple graph  $G$ , we obtain a general linear upper bound  $\text{reg}(I^{(n)}) \leq 2n + \text{ordmatch}(G) - 1$ , where  $\text{ordmatch}(G)$  is the ordered matching number of  $G$ .

**Bernd Ulrich**

***Ideals in the linkage class of a complete intersection: recent results and conjectures***

Linkage (or liaison) has been used since the nineteenth century to study and classify curves in projective three-space and, more generally, varieties in projective space or homogeneous ideals in polynomial rings. Of particular importance have been licci ideals, ideals that can be linked to a complete intersection in a finite number of steps. It was known that the Castelnuovo-Mumford regularity of a licci ideal forces a very strict upper bound for the initial degree of the ideal. Now, in joint work with Craig Huneke and Claudia Polini, we conjecture that it also provides a strong upper bound for the number of generators of the ideal, and we prove this conjecture in many cases. In addition, we provide new sufficient conditions for an ideal to be licci, for classes of ideals of height three and for ideals containing a maximal regular sequence of quadrics. In the talk we will also explain connections to recent work by Guerrieri, Ni, Weyman and by Jelisiejew, Ramkumar, Sammartano.

**Jugal Verma**

***Hilbert-Kunz multiplicity of powers of an ideal***

Let  $(R, \mathfrak{m})$  be a  $d$ -dimensional local ring,  $I$  be an  $\mathfrak{m}$ -primary ideal and let  $R$  have prime characteristic  $p$ . K.-I. Watanabe and K.-I. Yoshida investigated the Hilbert-Kunz multiplicity of powers of  $I$  in terms of Hilbert coefficients of  $I$  and its Frobenius powers  $I^{[q]}$

where  $q = p^n$ . It was proved by V. Trivedi that if  $I$  is zero-dimensional graded ideal of a standard graded ring  $R$  of dimension  $d$  over a field, then  $L_1(I) = \lim_{q \rightarrow \infty} e_1(I^{[q]})/q^d$  exists. Illya Smirnov proved Trivedi's result for  $\mathfrak{m}$ -primary ideals of all local rings. Smirnov asked if  $L_k(I) = \lim_{q \rightarrow \infty} e_k(I^{[q]})/q^d$  exists for  $k = 2, 3, \dots, d$  and whether the HK multiplicity of  $I^n$  for all large  $n$  is given by the formula

$$e_{HK}(I^n) = \sum_{k=0}^d (-1)^k L_k(I) \binom{n+d-k-1}{d-k}.$$

Smirnov also conjectured that ideals of reduction number one can be characterised in terms of the HK multiplicity. I will report on joint works with Kriti Goel, Arindam Banerjee and Shreedevi Masuti, Marilina Rossi and Alessandro De Stefani, which provide partial answers to Smirnov's questions.

## **Volkmar Welker**

### ***On subadditivity of shifts***

In this talk we explain two constructions on the simplicial chain complex of a poset:

- (i) the shuffle product (in case the poset is a lattice)
- (ii) the synor complex, which is a subcomplex of the simplicial chain complex whose chains have a well controlled structure. The synor complex is also shown to give rise to minimal free resolutions of monomial ideals.

Then we show how (i) and (ii) can be used to answer a question by Avramov, Conca and Iyengar on the subadditivity of shifts in the minimal free resolution in the case of monomial ideals.

## **Guangjun Zhu**

### ***Edge ideals of some edge-weighted graphs***

In this talk, we classify all normal edge-weighted graphs and give exact formulas or some bounds for the regularity and the depth of powers of the edge ideals of some edge-weighted graphs.