Abstract:

The "Eisenstein ideal" concept follows from congruences between cusp forms and Eisenstein series. The primordial and illustrious example resides in Ramanujan's congruence: if one expands $q\prod_{m=1}^{\infty}(1-q^m)^{24}$ as a series $\sum_n \tau(n)q^n$, $\tau(n)$ turns out to be congruent to$\sum_{d|n} d$ modulo 691

As part of the theory of

1. Modular forms, these notions have been invaluable for both

2. Arithmetic Geometry and

3. Algebraic Number Theory

The arithmetical geometry side has roots that far predates Ramanujan in the method of descent. We will highlight some of the developments throughout the 20th century that led to the crowning achievement in Mazur's torsion theorem of 1977. This side of the story ended in the mid 1990's. The algebraic number theory side is a continuing story. We will describe how it led to current research.