Abstract

We prove the reducibility of a class of quasi-periodically time dependent linear operators, which are derived from linearizing the dispersive third order Benjamin-Ono equation on the circle at a small amplitude quasi-periodic function, with a diophantine frequency vector $\omega \in \mathcal{O}_0 \subset \mathbb{R}^\nu$. It is shown that there exists a set $\mathcal{O}_\infty \subset \mathcal{O}_0$ of asymptotically full Lebesgue measure such that for any $\omega \in \mathcal{O}_\infty$, the operators can be reduced to the ones with constant coefficients by some linear transformations depending on time quasi-periodically. These transformations include a change of variable induced by a diffeomorphism of the torus, the flow of some PDEs and a pseudo-differential operator of order zero. We first reduce the linearized operator of order three to the one with constant coefficients plus a remainder of order zero, and then a perturbative reducibility scheme is performed. The major diffculties encountered are brought by the non-smooth character of the dispersive relation in view of the presence of the Hilbert operator $\mathcal{H}$. We look for several appropriate transformations which are real, reversibility-preserving and satisfy the sharp tame bounds which are used for the reducibility. This work will be the first fundamental step in proving the existence of time quasi-periodic solutions for the dispersive third order Benjamin-Ono equation.